

Research on the Public Transport Network Based on Complex Network

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Abstract

The public transport infrastructure of a city is one of the most important indicators of its economic growth and development. Here we investigate the statistical properties of the public transport network in Shenyang to explore its various properties based on complex network theory. The statistical properties of the public transport system consist of the degree of a node, the average shortest path length, the clustering coefficient of a node, the average clustering coefficient, and the degree distribution. In contrast with the small world evolution model, we find that the public transport system of Shenyang, a network of public transportation routes connected by bus links, is a small-world network characterized by a Poisson degree distribution. Simulation results show that the public transport network exhibits small world behavior with $N=148$.

Keywords: public transport network, complex network, small-world network

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1. Introduction

Transportation infrastructures are of crucial importance to the development of a country. They support movement of goods and people across the country, thereby driving the national economy [1]. Roadways, railways, and airways are the major means of transport in China. Understanding of these transportation systems is important for reasons of policy, administration and efficiency.

The intricate structure of interactions of many natural and social systems has been object of intense research in the new area of complex networks. Most of the effort in this area has been directed to find the topological properties of real world networks and understand the effects that these properties cast on dynamical processes taking place on these complex networks. For instance, the small-world characteristic, where each node of the network is only a few connections apart from any other, permits a quick spreading of information through the network, being fundamental in processes of global coordination and feedback regulation [2].

During the past few years, since the explosion of the complex network science that has taken place after works of Watts and Strogatz [3] as well as Barabasi and Albert [4, 5] a lot of real-world networks have been examined, such as Internet, WWW, world-wide airport network, commutation networks, electric networks, food webs, cell metabolism, scientific research cooperation relations, and citation networks [6-8]. The pioneering work of Watts and Strogatz [3] opened a completely new field of research. Its main contribution was to show that many real-world networks have properties of random graphs and properties of regular low dimensional lattices. A model that could explain this observed behavior was missing and the proposed "small-world" model of the authors turned the interest of a large number of scientist in the statistical mechanics community in the direction of this appealing subject [9]. A model was proposed for the evolution of weighted evolving networks in an effort to understand the statistical properties of real-world systems. The topology of these systems was found to be having small-world network features and a two-regime power-law degree distribution.

Despite this, at the beginning little attention has been paid to transportation networks-mediums as much important and also sharing as much complex structure as those previously listed. However, during the last few years several public transport systems have been investigated using various concepts of statistical physics of complex networks [10].

In this paper, we have studied a part of data for the public transport system in Shenyang and we have analyzed their nodes degrees, the average shortest path length, the clustering coefficient, the average clustering coefficient, and the degree distribution. In contrast with the small world evolution model, our analysis shows that the public transport system has small-world network features and has a Poisson distribution.

2. The Model of Public Transport Network

To analyze various properties of the public transport system one should start with a definition of a proper network topology. The idea of the space L and P, proposed in a general form in [11] and used also in [12] is presented at Figure 1. The first topology (space L) consists of nodes representing bus, tramway or underground stops and a link between two nodes exists if they are consecutive stops on the route. The node degree k in this topology is just the number of directions (it is usually twice the number of all public transport system routes) one can take from a given node while the distance l equals to the total number of stops on the path from one node to another [10].

Although nodes in the space P are the same as in the previous topology, here an edge between two nodes means that there is a direct bus, tramway or underground route that links them. That is to say, if a route A consists of nodes $a_i (i=1,2,\dots,n)$, then in the space P the nearest neighbors of the node a_1 are a_2, a_3, \dots, a_n . Consequently the node degree k in this topology is the total number of nodes reachable using a route and the distance can be interpreted as a number of transfers one has to take to get from one stop to another.

Another idea of mapping a structure embedded in two-dimensional space into another, dimensionless topology was used, where a plan of the city roads has been mapped into an information city network. In the last topology a road represents a node and an intersection between roads - an edge, so the network shows information handling that has to be performed to get oriented in the city [10].

In the paper, we consider the second topology (space P). Such an several other types of network systems: Internet, railway or airport networks.

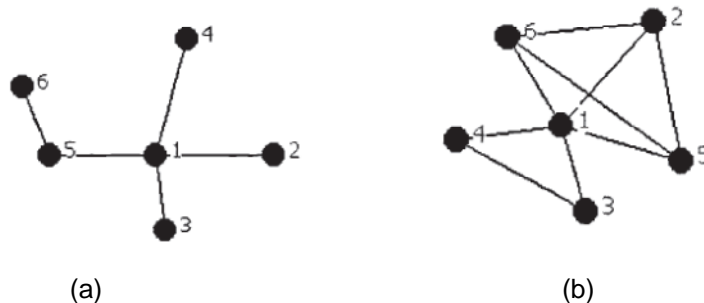


Figure 1. Explanation of the Space L (a) and the space P (b)

3. Topological Analysis of the Public Transport Network

Degree of a node is the number of nodes it is directly connected to. Degree of a node i is defined as:

$$k_i = \sum_{j=1}^N a_{ij} \quad (1)$$

In a directed network, in-degree (out-degree) of a node is the number of in-coming (out-going) links. In the public transport network of Shenyang, in-degree (k_i^{in}) and out-degree (k_i^{out}) of the public transport stand for the number of bus terminating-into and number of bus originating-from, respectively. We observe that for a very large number of nodes in the public transport network $k_i^{in} = k_i^{out}$.

In a network, the distance between two nodes, labeled i and j respectively, is defined as the number of edges along the shortest path connecting them. The average shortest path length L of the network, then, is defined as the mean distance between two nodes, averaged over all pairs of nodes.

The average shortest path length (L) for a directed network with N nodes is defined as:

$$L = \frac{1}{N(N-1)} \sum_{i,j=1;i \neq j}^N L_{ij} \quad (2)$$

Where L_{ij} is the shortest path length from node i to j .

In terms of network topology, clustering, also known as transitivity, is a typical property of acquaintance networks, where two individuals with a common friend are likely to know each other [7]. In terms of a generic graph G , transitivity means the presence of a high number of triangles. The clustering coefficient (C_i) of a node is defined as the ratio of number of links shared by its neighboring nodes to the maximum number of possible links among them. In other words, C_i is the probability that two nodes are linked to each other given that they are both connected to i [13].

There are two definitions of the clustering coefficient. Here, we adopt the widely used definition given by Watts and Strogatz [3].

$$c_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}. \quad (3)$$

A quantity c_i (the local clustering coefficient of node i) is first introduced, expressing how likely $a_{jm} = 1$ for two neighbors j and m of node i . Its value is obtained by counting the actual number of edges (denoted by e_i) in G_i (the subgraph of neighbors of i). The local clustering coefficient is defined as the ratio between e_i and $k_i(k_i - 1)$, the maximum possible number of edges in the network [2]:

$$c_i = \frac{2e_i}{k_i(k_i - 1)} = \frac{\sum_{j,m} a_{ij} a_{jm} a_{mi}}{k_i(k_i - 1)}. \quad (4)$$

For vertices with degree 0 or 1, for which both numerator and denominator are zero, they are defined as $c_i = 0$.

Then the clustering coefficient for the whole network is the average.

$$C = \langle c \rangle = \frac{1}{N} \sum_{i=1}^N c_i \quad (5)$$

Where $0 \leq c_i \leq 1$, $0 \leq C \leq 1$ and c_i is the clustering coefficient of node i .

From the above definitions, it can be seen that C is a measure of the relative number of triangles, and is strictly in the interval $[0, 1]$, with the upper limit attained only for a fully connected graph. In a social acquaintance network, for example, $C = 1$ if everyone in the network knows each other. In addition, it has to be noted that even though the BA model successfully explains the scale-free nature of many networks, it has $C = 0$ and thus fails to describe networks with the high clustering, such as social networks.

Here we only analyzed a part of data for the public transport system in Shenyang. Moreover we chose numbers of nodes in the public transport network of Shenyang $N = 148$.

One should notice that other surveys exploring the properties of transportation networks have usually dealt with smaller numbers of vertices, such as $N = 76$ for U-Bahn network in Vienna [12], $N = 124$ in Boston Underground Transportation System [14] or $N = 128$ in Airport Network of China [15].

Regular lattices are clustered, but do not exhibit the small-world effect in general. On the other hand, random graphs show the small-world effect, but do not show clustering. Thus, it is not surprising to see that the regular lattice model and the ER random model both fail to reproduce some important features of many real networks. After all, most of these real-world networks are neither entirely regular nor entirely random. The reality is that people usually know their neighbors, but their circle of acquaintances may not be confined to those who live right next door, as the lattice model would imply. On the other hand, cases like links among Web pages on the WWW were certainly not created at random, as the ER process would expect [16].

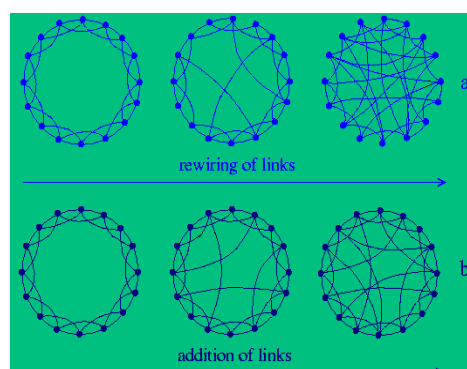
Aiming to describe a transition from a regular lattice to a random graph, Watts and Strogatz [3] introduced an interesting small-world network model, referred to as WS small-world model. The WS small-world model algorithm as follows:

1) Start with order: Begin with a nearest-neighbor coupled network consisting of N nodes arranged in a ring, where each node i is adjacent to its neighbor nodes, $i = 1, 2, \dots, k/2$, with k being even.

2) Randomization: Randomly rewire each edge of the network with probability p ; varying p in such a way that the transition between order ($p = 0$) and randomness ($p = 1$) can be closely monitored.

The work on WS small-world networks has started an avalanche of research on new models of complex networks, including some variants of the WS model. A typical variant was the one proposed by Newman and Watts [17], referred to as the NW small-world model lately. In the NW model, one does not break any connection between any two nearest neighbors, but instead, adds with probability p a connection between a pair of nodes. Likewise, here one does not allow a node to be coupled to another node more than once, or to couple with itself.

With $p = 0$, the NW model reduces to the original nearest-neighbor coupled network, and if $p = 1$ it becomes a globally coupled network. The NW model is somewhat easier to analyze than the original WS model because it does not lead to the formation of isolated clusters, whereas this can indeed happen in the WS model. For sufficiently small p and sufficiently large N , the NW model is essentially equivalent to the WS model. Today, these two models are together commonly termed small-world models for brevity [16]. The WS model and the NW model can be generated as follows (Figure 2) [16].



(a) The WS small-world model; (b) The NW small-world model

Figure 2. The Small-world Model

According to the NW small-world model with $p = 0.4, k = 4$ and $N = 148$, the average shortest path length and clustering coefficient, 1.5215 and 0.4587, respectively. When the NW small-world model with $p = 0.2, k = 4$ and $N = 148$, the average shortest path length and clustering coefficient, 1.7353 and 0.2576, respectively.

Small-world networks are characterized by a very small average shortest path length (L) and a high average clustering coefficient (C). Shortest path length from node i to j , L_{ij} , is the number of bus needed to be taken to go from i to j by the shortest route.

We found the average shortest path length of the public transport network to be $L = 1.9392$, which is of the order of that of a random network of same size and average degree. The average clustering coefficient of the public transport network was found to be $C = 0.5385$, which is an order of magnitude higher than that of the comparable random network. These two properties indicate that the public transport network is a small-world network. ANI [1] has also been found to be small-world networks. In particular, ANI has average shortest path length and clustering coefficient, 2.2593 and 0.6574, respectively.

Table 1. The Statistical Properties of the Various Networks with $N=148$

The statistical properties	the public transport network	NW small-world network with $p=0.4, k=4$	NW small-world network with $p=0.2, k=4$
the average shortest path length(L)	1.9392	1.5215	1.7353
the average clustering coefficient(C)	0.5385	0.4587	0.2576

Table 1 shows the statistical properties of the various networks with $N=148$, which consists of the public transport network and NW small-world network with different parameters. Our analysis shows that while the public transport network is similar to the ANI [1] in some aspects, it has differences in some features as reflected in its network parameters. We find that the public transport network has small-world network features, in contrast to NW small-world network.

Since the public transport network is a small network, we analyze the cumulative degree distribution. We find that the cumulative degree distribution of the public transport network follows a Poisson as seen in Figure 3. As we can see from Figure 4 and Figure 5, the cumulative degree distribution of the NW small-world model also follow a Poisson.

Figure 3 shows typical plots for degree distribution in the public transport network of Shenyang. Let us notice that the number of nodes of degree $k = 1$ is smaller as compared to the number of nodes of degree $k = 2$ since $k = 1$ nodes are ends of transport routes. Still some nodes (hubs) can have a relatively high degree value (in some cases above 20) but the number of such vertices is very small. Degree distributions obtained for the public transport network is Poisson distribution as the NW small-world model is.

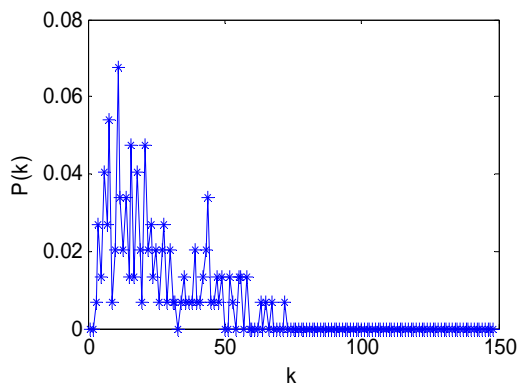


Figure 3. Degree Distribution in the Public Transport Network of Shenyang

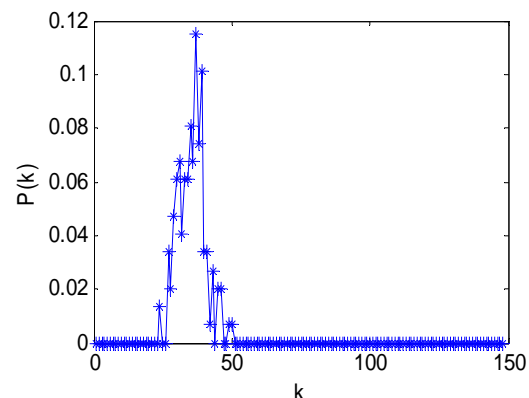


Figure 4. Degree Distribution in NW Small-world Network with $p=0.2, k=4$ and $N=148$

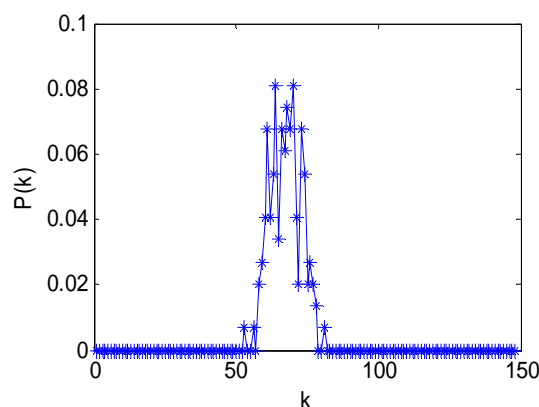


Figure 5. Degree Distribution in NW Small-world Network with $p=0.4$, $k=4$ and $N=148$

4. Conclusion

In this study we have collected and analyzed a part of data for the public transport network in Shenyang. Sizes of this network is $N=148$. Using the concept of different network topologies we show that in the space P , where distances are measured in numbers of passed bus/tramway stops. Its topology was found to be having small-world network features. Many of our results are similar to features observed in other works regarding transportation networks: underground, railway or airline systems [1], [10-11], [14-15]. All such networks tend to share small-world properties.

Acknowledgements

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