# Measurement Algorithm of Two-Dimensional Wind Vector using Ultrasonic Transducers 

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#### Abstract

Ultrasonic wind speed measurement technique obtains more and more extensive use in the field of Meteorology and Environmental Protection etc. When designing an ultrasound wind vector instrument, synthesizing wind vector information, containing wind velocity vector and wind angle, is of prime importance to accomplish accurate measurement. This paper presents synthesis algorithm of ultrasonic 2D (two-dimensional) wind vector based on phase difference wind detection method, establishes mathematical model of wind speed, ultrasonic speed and phase difference, focuses on the analysis of the quadrant critical problem in the process of synthesizing wind angle. And the ultrasonic phase difference wind speed and detection method proposed in this paper overcomes most disadvantages, e.g. complex circuit, troublesome signal processing etc., which exist in traditional time difference ultrasonic wind detection method. Through software emulation, the mathematical model established in this paper could calculate wind speed and direction precisely, based on 2D wind speed component.


Keywords: ultrasonic, two-dimensional, 2D, wind speed, wind angle
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## 1. Introduction

With the progress of science and technology, ultrasonic wind speed measurement technology has developed rapidly. Compared to traditional mechanical measurement, the ultrasonic wind measurement technique has many advantages including: executable micro air velocity measurement, no upper limit of theoretical measurement, no impact on wind field measurement, without impact of the changes of airflow elements, no rotating components, no mechnical inertia, small consumption of machine, dispensation with starting threshold of wind velocity, convenient maintenance. Nowadays, wind velocity measurement plays an important part in environment monitoring, bridge and tunnel monitoring, navigation and aviation monitoring, wind power generation, military application etc [1]. At present ultrasonic wind speed measurement technique has proved to be an important detection technique in the field of wind velocity measurement [2]. This work has an importance in the wind energy sector, especially in the advancement of instrumentation and methodologies for wind speed measurement and wind direction.

Based on vector analysis method, ultrasonic wind speed measurement techniques capitalize on the ultrasonic signal bearer of gas flow information when ultrasonic is traveling through air to realize measurement [3]. The focus on using this kind of techniques of which the most widely used one is time difference wind speed measurement method, is to get the flow information carrying on ultrasonic signal precisely. The fundamental of time difference wind speed measurement is reflecting wind speed through the time difference while ultrasonic pulse is speeding between headwind and downwind under the same acoustic distance. Obtaining time difference is a decisive factor of wind speed measurement, and determines the measurement accuracy directly. With respect to direct method of time difference measurement, phase difference measurement method that pertains to time measurement has many advantages, such as low circuit cost, easy to realize hardware and software etc. Based on phase difference measurement method, measurement algorithm of 2D wind vector in this paper is designed [4].

## 2. Ultrasonic 2D wind vector measurement principle

Sensors whose configuration chosen is triangular structure use three ultrasonic TR (Transmitter-Receiver) transducers. Configuration form is shown in Figure 1.


Figure 1. Transducers triangular configuration

Assuming transducer $A$ is used as launch probe, and receiving probes are $B$ and $C$. At some point, when wind direction shows as $V$, wind speed value on the direction of $B C$ can be found by means of analysis on the triangular structure. Similarly, when transducer B is used as launch probe, and receiving probes are $A$ and $C$, wind speed value on the direction of $A C$ appears. That is to say, three probes take turns launching and receiving, and three wind speed value can be gained. Any wind (speed and direction are uncertain) can be divided into 3 components on the direction of triangle's edges, so synthesize 3 vectors in order to obtain ultimate wind vector information.

For the synthesis of wind vector, it needs the positive direction of wind speed, so instrument measuring positive direction should be defined which is generally true north. Table 1 lists the configuration of transmitter-receiver of each probe and the definition of wind components on each triangle's edge.

Table 1. Configuration of transmit-receive and positive direction of wind component

| Transmitter | Receiver | Positive direction |
| :---: | :---: | :---: |
| A | B,$~ C$ | $\mathrm{~B} \rightarrow \mathrm{C}$ |
| B | A C | $\mathrm{C} \rightarrow \mathrm{A}$ |
| C | A, B | $\mathrm{A} \rightarrow \mathrm{B}$ |

## 3. Algorithm of synthetizing wind vector

Measurement of wind vector includes: measuring 3 wind vector components on the direction of triangle's edges, each wind component taking vector synthesis to obtain wind speed value and the synthesis of the actual wind direction.

### 3.1 Three wind vector components on edges

As an example for analysis, this paper takes that transducer C is used as launch probe, and receiving probes are $A$ and $B$. Assuming current wind vector is $F$ shown as figure 2 of which the component on edge $A B$ is $V_{a b}$.


Figure 2. Synthesis of any wind vector $F$ and its component $V_{a b}$

Each edge of the equilateral triangle is $L$, $t_{1}$ is the time required for ultrasonic wave from the launch head $C$ to receive head $A, t_{2}$ is the time required for ultrasonic wave from $C$ to $B, c c$ is the ultrasonic propagation velocity in no wind environment and angle of equilateral triangle $\theta$ equals to $60^{\circ}$.

Based on the disassembly graph, a formula can be written as:

$$
\left\{\begin{array}{l}
t_{1}=\frac{L}{c c-V_{\mathrm{ab}} \cdot \cos \theta}  \tag{1}\\
t_{2}=\frac{L}{c c+V_{\mathrm{ab}} \cdot \cos \theta} \\
\Delta t=t_{1}-t_{2}
\end{array}\right.
$$

When $V_{\mathrm{ab}}$ is small and $V_{\mathrm{ab}} \ll c c$,

$$
\begin{equation*}
\Delta t=\frac{2 L \cdot V_{\mathrm{ab}} \cdot \cos 60^{\circ}}{c c^{2}}=\frac{L \cdot V_{\mathrm{ab}}}{c c^{2}} \tag{2}
\end{equation*}
$$

Then:
$V_{\mathrm{ab}}=\frac{\Delta t \cdot c c^{2}}{L}$
According to the derivation above, the equation between $V x\left(V x\right.$ takes $V_{a b}, V_{b c}$ or $\left.V_{a b}\right)$ and $L, c c, \Delta t$ is shown as:

$$
\begin{equation*}
V x=\frac{\Delta t \cdot c c^{2}}{L} \tag{4}
\end{equation*}
$$

Where $L$ and $c c$ are known, and $\Delta t$ is the time difference signal we want to measure. The form of $\Delta t$ performing on two received signals is phase deviation between these two signals, so phase detection circuit is designed to convert $\Delta t$ into phase difference. According to $\Delta \varphi=2 \pi f \cdot \Delta t$, it is able to get $\Delta t$ as long as $\Delta \varphi$ has been detected.

When three probes transmit and receive alternately, wind speed component in the direction of the connection between two receiving probes is found each turn. Specific names related to wind speed components in this paper are listed in Table 2.

Table 2. Names of wind speed components

| Transmitter | Receiver | Untested wind speed component |
| :---: | :---: | :---: |
| A | B, C | $V_{\mathrm{bc}}$ |
| B | A, C | $V_{\mathrm{ca}}$ |
| C | A, B | $V_{\mathrm{ab}}$ |

### 3.2 Wind speed value

Wind vector $F$ and its components is shown in figure 3 , in which wind speed value $|F|$ is named $V$. According to projection method, three components of the wind vector $F$ can be gotten along the triangle three side of the equilateral triangle $A B C$ by decompositing $F$.


Figure 3-a Decomposition of wind vector $F$


Figure 3-b Two vectors synthesize wind vector $F$

Figure 3. Components of wind vector $F$

Take each component of $F$ respectively named as $V_{a b}, V_{b c}, V_{c a}\left(V_{a b}\right.$ is the component along the edge AB of the triangle, of which the positive direction definition is from A to B . The $V_{b c}, V_{\text {ca }}$ can be done in the same manner). As is shown in figure 3-a (At this moment, assume the wind vector $F$ ). Based on the size of each side of the wind speed component and positive directions of each edge defined, detection of angle values between speed value $|\mathrm{F}|$ and each positive direction can be finished. Though synthesis calculation operates theoretically if only two components have been known, this paper acquires the average of three sets of data to calculate |F| in order to improve measurement accuracy.

Calculation of speed |F| should use projection synthesis. Take the synthetic process of $V_{\mathrm{ab}}$ and $V_{\mathrm{bc}}$ for example, to calculate $|\mathrm{F}|$, this process described as figure 3-b.

First, move the compenents $V_{a b}$ and $V_{b c}$ to point $A$--- one of the three apical angles of the equilateral triangle as shown.

Second, make the vertical line of $A B$ axis. And the connecting line between two points Vbc and V will cross the vertical line at D . So an equation can be written in the right triangle composited by $V_{b c}$ and $A D$. Let $A D=n$. Based on the theory of coangle, it's known that an angle equles to $\theta$, which is one angle of equilateral triangle $A B C$.

At last, $\angle \mathrm{DPQ}=60^{\circ}$ in the right triangle DPQ drawn in dotted line.
Use the Pythagorean Theorem in the right triangle composited by $V_{a b}, V$ and $(m+n)$.
From the above, a formula can be written as:

$$
\left\{\begin{array}{l}
V^{2}=V_{\mathrm{ab}}{ }^{2}+(m+n)^{2}  \tag{5}\\
m=V_{\mathrm{ab}} / \operatorname{tg} \theta=V_{\mathrm{ab}} / \operatorname{tg} 60^{\circ} \\
n=V_{\mathrm{bc}} / \operatorname{ctg} \theta=V_{\mathrm{bc}} / \operatorname{ctg} 60^{\circ}
\end{array}\right.
$$

Here, values of $V_{\mathrm{ab}}$ and $V_{\mathrm{bc}}$ can be obtained directly by measuring. Do algebraic equations and eliminate $m$ and $n$, then:

$$
\begin{equation*}
V=\frac{4}{3} \sqrt{V_{\mathrm{bc}}{ }^{2}+{V_{\mathrm{ab}}^{2}+V_{\mathrm{ab}} \cdot V_{\mathrm{bc}}}^{\text {a }} \text {. }} \tag{6}
\end{equation*}
$$

$V_{c a}$ and $V b c$ compose $V^{\prime}$, likewise, $V_{\mathrm{ca}}$ and $V_{\mathrm{ab}}$ compose $V^{\prime}$, so speed $|F|$ equals to $\left(V+V^{\prime}+V^{\prime}\right) / 3$, Where $V^{\prime}$ and $V^{\prime}$ are:

$$
\begin{align*}
& V^{\prime}=\frac{4}{3} \sqrt{V_{\mathrm{bc}}^{2}+V_{\mathrm{ca}}^{2}+V_{\mathrm{ca}} \cdot V_{\mathrm{bc}}}  \tag{7}\\
& V^{\prime}=\frac{4}{3} \sqrt{V_{\mathrm{ca}}{ }^{2}+V_{\mathrm{ab}}^{2}+V_{\mathrm{ab}} \cdot V_{\mathrm{ca}}} \tag{8}
\end{align*}
$$

## 4. Algorithm of synthetizing wind angle

To measure wind angle, positive directions of each triangle side and instrument measuring direction have been defined. Progress of synthetizing wind angle should take the relationship between positive directions of each triangle side and the measuring positive direction into account. Assuming $A \rightarrow B$ as the measuring positive direction, the angle can be calculated as:

$$
\begin{equation*}
\theta_{\mathrm{AB}}=\theta_{\mathrm{BC}}-120^{\circ}=\theta_{\mathrm{CA}}+120^{\circ} \tag{9}
\end{equation*}
$$

Where $\theta_{A B}$ is the angle between wind vector and $A B$ side, $\theta_{B C}$ is the angle between wind vector and $B C$ side, in the same way, $\theta_{C A}$ is the angle between wind vector and CA side.

Also takes the synthetic process of $V_{a b}$ and $V_{b c}$ for example, to calculate wind vector angle, analysis procedure shown as figure 4.


Figure 4. Two vectors synthesize wind vector angle

Based on figure 4, this paper gets:

$$
\begin{align*}
& \left\{\begin{array}{l}
V_{\mathrm{ab}}=V \times \cos (\beta) \\
V_{\mathrm{bc}}=V \times \cos \left(\beta-120^{\circ}\right)
\end{array}\right.  \tag{10}\\
& \left\{\begin{array}{l}
V_{\mathrm{ab}}=V\left[\cos \left(\beta-60^{\circ}\right) \cos 60^{\circ}-\sin \left(\beta-60^{\circ}\right) \sin 60^{\circ}\right] \\
V_{\mathrm{bc}}=V\left[\cos \left(\beta-60^{\circ}\right) \cos 60^{\circ}+\sin \left(\beta-60^{\circ}\right) \sin 60^{\circ}\right]
\end{array}\right.  \tag{11}\\
& V_{\mathrm{ab}}+\mathrm{V}_{\mathrm{bc}}=2 V \cos \left(\beta-60^{\circ}\right) \cos 60^{\circ}  \tag{12}\\
& V_{\mathrm{ab}}-V_{\mathrm{bc}}=-2 V \sin \left(\beta-60^{\circ}\right) \sin 60^{\circ} \tag{13}
\end{align*}
$$

$(12) \div(13):$

$$
\begin{equation*}
\operatorname{tg}\left(\beta-60^{\circ}\right)=\frac{V_{\mathrm{bc}}-V_{\mathrm{ab}}}{V_{\mathrm{ab}}+\mathrm{V}_{\mathrm{bc}}} \times \frac{1}{\sqrt{3}} \tag{14}
\end{equation*}
$$

Deriving this formula:

$$
\begin{equation*}
\beta=\arctan \left(\frac{V_{\mathrm{bc}}-V_{\mathrm{ab}}}{V_{\mathrm{ab}}+V_{\mathrm{bc}}} \times \frac{1}{\sqrt{3}}\right)+60^{\circ} \tag{15}
\end{equation*}
$$

It means that, $\beta$ is the angle formed by wind vector and instrument measuring direction. In the same way, $\beta^{\prime}$ and $\beta^{\prime \prime}$ composed by the other two sets of data, as shown in table 3.

Table 3. Synthetic component of the wind vector and synthetic angle comparison table

| Component 1 | Component 2 | Formula of synthesizing angle |
| :---: | :---: | :---: |
| $V_{\mathrm{ab}}$ | $V_{\mathrm{bc}}$ | $\beta=\arctan \frac{V_{\mathrm{bc}}-V_{\mathrm{ab}}}{\sqrt{3}\left(V_{\mathrm{ab}}+V_{\mathrm{bc}}\right)}+60^{\circ}$ |
| $V_{\mathrm{bc}}$ | $V_{\mathrm{ca}}$ | $\beta^{\prime}=\arctan \frac{V_{\mathrm{ca}}-V_{\mathrm{bc}}}{\sqrt{3}\left(V_{\mathrm{ca}}+V_{\mathrm{bc}}\right)}$ |
| $V_{\mathrm{ca}}$ | $V_{\mathrm{ab}}$ | $\beta^{\prime \prime}=\arctan \frac{V_{\mathrm{ca}}-V_{\mathrm{ab}}}{\sqrt{3}\left(V_{\mathrm{ab}}+V_{\mathrm{ca}}\right)}-60^{\circ}$ |

As the range of $y=\arctan (x)$ is $(-\pi / 2, \pi / 2)$, calculated angle will be deviate from true angle for the case that wind speed angle is larger than $90^{\circ}$, so the utility function chooses atan2( $\mathrm{y}, \mathrm{x}$ ).

Return value of this four-quadrant arctangent function atan2 $(y, x)$ is the arc tangent value calculated by $x$ given and coordinate value $y$. Arc tangent angle equals to the angle formed by axis $x$ and the line connecting the origin and the given coordinates points ( $x, y$ ). Range of the result expressed as radian is $[-\pi, \pi)$. Using atan2 $(y, x)$ to solve angle, it can gain the unique wind vector value located in all quadrant.

Using this function at $\pm 180^{\circ}$, one case may exists as shown in Figure 5: Angles composed by three sets of data are in close proximity to negative x-axis axle. If this time the wind is located in the second quadrant, the calculated value will be less than $180^{\circ}$ but close to $180^{\circ}$. If this time the wind is located in the third quadrant, the calculated value will be larger than $-180^{\circ}$ but close to $-180^{\circ}$, as shown in figure 5 .


Figure 5. Special cases while wind angle is solving

Assuming true speed angle is $179^{\circ}$, measurand $\beta=178^{\circ}, \beta^{\prime}=174^{\circ}$, the accurate value of $\beta^{\prime \prime}$ is: $3 \times 179^{\circ}-\beta-\beta^{\prime}=185^{\circ}$, but for atan2( ), the calculated number of $\beta^{\prime \prime}$ will be $-175^{\circ}$ for function atan2( ). Thus, it will result in computational error directly from using the direct value of $\beta, \beta^{\prime}$ and $\beta^{\prime \prime}$ to calculate the average angle.

Aiming at problems nearby $\pm 180^{\circ}$, this paper translates calculated angles into $0^{\circ} \sim$ $360^{\circ}$ before data procession. The basis of signal procession is: Theoretically, the difference of any two of $\beta, \beta^{\prime}$ and $\beta^{\prime \prime}$ meets $\Delta \beta \in\left(-180^{\circ}, 180^{\circ}\right)$, so the procession of angle is needed in case that the difference of two angle doesn't belong to this range.

Processing idea: Take out the largest one of three angles $\beta, \beta^{\prime}$ and $\beta^{\prime \prime}$, assuming it is, and calculate the differences of $\beta$ and each of the rest $\beta^{\prime}$ and $\beta^{\prime}$ respectively. If two differences are larger than $180^{\circ}$ or less than $-180^{\circ}$ at the same time, let $x=2$ and $k=1$. If only one difference is larger than $180^{\circ}$ or less than $-180^{\circ}$, let $x=1$ and $k=1$. If two differences all belong to $\left(-180^{\circ}, 180^{\circ}\right)$ and $\beta, \beta^{\prime}, \beta^{\prime \prime}$ are all larger than 0 , let $x=0$ and $k=1$. If two differences all belong to $\left(-180^{\circ}, 180^{\circ}\right)$ and $\beta, \beta^{\prime}, \beta^{\prime}$ ' are all less than 0 , let $x=0$ and $k=-1$.

Calculate $\theta=k \times\left(\beta+\beta^{\prime}+\beta^{\prime}+360^{\circ} \times x\right) \div 3$, change $\theta$ into the range $\left(0 \sim 360^{\circ}\right)$, and we can gain the final wind vector angle.

## 5. Comparation of simulation data

Use C programming language to realize the algorithms above. Give five sets of wind speed component manually, and calculate the true angle and speed of wind vector on the basis of projection synthesis method. Then each simulation value with software simulation method of component values is given, and the real value can be compared to them to discuss accuracy of this algorithm.

The simulation results confirm that using the algorithm introduced in this work, the real speed and angle value of wind vector and the theoretical calculation result are with fine consistency. Comparison table of wind speed and angle is as table 4 and 5.

Table 4. Synthetic component of the wind speed and synthetic speed comparison table

| Data Name | Set 1 | Set 2 | Set 3 | Set 4 | Set 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{\mathrm{ab}}(\mathrm{m} / \mathrm{s})$ | 0 | -5.0 | -20.0 | 0 | 8.0 |
| $V_{\mathrm{bc}}(\mathrm{m} / \mathrm{s})$ | 8.6 | -5.0 | 10.0 | -17.3 | 8.0 |
| $V_{\mathrm{ca}}(\mathrm{m} / \mathrm{s})$ | -8.7 | 10.0 | 10.0 | 17.3 | -16.0 |
| Calculated $V(\mathrm{~m} / \mathrm{s})$ | 9.96 | 10.0 | 20.0 | 19.93 | 16.0 |
| Simulated $V(\mathrm{~m} / \mathrm{s})$ | 9.9 | 10.0 | 20.0 | 19.9 | 16.0 |

Table 5. Synthetic component of the wind angle and synthetic angle comparison table

| Data Name | Set 1 | Set 2 | Set 3 | Set 4 | Set 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\beta\left({ }^{\circ}\right)$ | 90 | -120 | 178 | -90 | 60 |
| $\beta^{\prime}\left({ }^{\circ}\right)$ | 91 | -122 | 179 | -92 | 59 |
| $\beta^{\prime \prime}\left({ }^{( }\right)$ | 89 | -118 | -179 | -88 | 61 |
| Calculated $\theta\left({ }^{\circ}\right)$ | 90 | 240 | 179 | 270 | 60 |
| Simulated $\theta\left({ }^{\circ}\right)$ | 90 | 240 | 179 | 270 | 60 |

## 6. Factors Influencing Measurement and Solutions

Factors taken influencing measurement in consideration include temperature, humidity and impurity in propagation medium. To achieve accurate wind speed value, compensation actions should be done before the calculating progress mentioned in this paper.

### 6.1 Attenuation

Attenuation in propagation process is shown in this equation:

$$
\begin{equation*}
I=I_{0} e^{-\alpha x} \tag{10}
\end{equation*}
$$

$I$ - Sound intensity, $I_{0}$ - Sound intensity in initial
$\alpha$ - Attenuation coefficient, $x$ — Medium thickness
Attenuation coefficient refers to scattering attenuation coefficient $\alpha_{s}$ and absorption attenuation coefficient $\alpha_{a}$. The frequency $f$ of ultrasonic should not be high in order to reduce the attenuation, as $\alpha_{s}$ is into direct proportion relationship with $f^{4}$ and $\alpha_{a}$ is into direct proportion relationship with $f^{2}$.

### 6.2 Temperature

Calculating the derivative or differential of Eq.(4), there is

$$
\begin{equation*}
d V x=\frac{2 c c}{L} \cdot \Delta t d c c+\frac{c c^{2}}{L} d \Delta t-\frac{\Delta t \cdot c c^{2}}{L^{2}} d L \tag{11}
\end{equation*}
$$

The propagation of ultrasonic is $c c=331.5+0.607 \times \mathrm{T}$ where T means temperature. It can be calculated by the (11) that the relative error brought by the change of the temperature is about $0.18 \%$ by $1^{\circ} \mathrm{C}$, while the product design allows for the measurement error within $2 \%$. As
for this, ambient temperature tested by a temperature sensor participates in calculation to enhance the precision of measurement.

### 6.3 Time of Flight (TOF)

Time of flight refers to the time that ultrasonic wave propagates in medium. TOF influences the measurement feedback from sensors directly, and largely determines the accuracy of measurement.

This paper presents an ultrasonic 2D wind vector detection method based on phase difference. In the three-sensor-configuration, TOF is equal for the two receivers when the ultrasonic wave carrying wind information sends from the transmitter to each receiver. According to the above analysis, it's known that the two signals received are in the upwind and downwind. By high-precision phase comparison circuit, the lag between the ultrasonic which was received by two ultrasonic receiving sensors during the traveling times in the upwind and downwind is converted to the phase difference. This process can be understood as differential signal processing. So TOF is dealt. After that, the wind velocity is measured.

## 7. Conclusion

This paper presents an ultrasonic 2D wind vector detection method based on phase difference, and takes projection synthesis method to use different wind velocity component to synthesize wind speed and angle, relationship of which has been stated in this work. Simulation results show that the established mathematical model existing among the wind speed, speed of sound and the phase difference can calculate speed and direction of wind vector effectively. So ultrasonic wind vector anemometer designed based on this method can provide more accurate wind vector information for environmental monitoring, meteorological monitoring, marine applications, military applications occasions.

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