3D Reconstruction of Patient-specific Femurs Using Coherent Point Drift

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Abstract

This paper dealt with the problem that the overlapping digital radiographs couldn't reflect the 3D space information of the patient-specific femur in the orthopaedic surgery diagnosis. A 2D-3D non-rigid registration method based on Coherent Point Drift was proposed to realize the 3D reconstruction of the patient-specific femur before the surgery, which used bi-planar digital radiographs of the patient-specific femur and the CT volume data of a generic femur. With the advantages of low cost, fast imaging speed and little radiation to the patients and doctors, this method provided more effective 3D imaging information for the femur diagnosis and preoperative plans. The registration experiments showed that the proposed method recovered the 3D model and the pose of the patient-specific femur effectively with a fast, accurate and robust registration result, which had satisfied the needs of clinical application.

Keywords: 2D-3D registration, gaussian mixture model, CPD algorithm, femur, preoperative orthopedic surgical planning

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1. Introduction

Preoperative planning in orthopedic surgery is essential to identify the optimal surgical considerations for each patient-specific case. 3D reconstruction of bones plays an important role in computer-assisted diagnosis and surgery [1]. Conventional diagnosis and preoperative planning of femur reduction and fixation rely primarily on 2D radiographs (films or digital radiographs, which we called DR). The surgeon tries to imagine a 3D vision of the bone and to plan the operation in his or her mind. Imaging modalities such as CT have the ability to provide direct 3D volumetric images. However, the use of such imaging is restricted to a minority of complex procedures due to constraints placed by cost, availability and risks posed by unwarranted detailed imaging. To overcome the inefficiency of the methods mentioned above, researchers began to develop a system to build 3D models from 2D radiographs for 3D reconstruction.

In the 1990s, some studies were proposed for 3D model reconstruction using two different angle X-ray images. In the early relevant works, the bi-planer images were used to realize the 3D reconstruction. Based on the traditional bi-planar X-ray films, L Caponetti [2] reconstructed the femur skeleton model of 3D pose by using a polygon mesh and 3D B-spline interpolation method. B Nikkhade-Dehkordi et al. [3] made the assumptions that the bone shape was axisymmetric and the reconstruction of 3D model was synthesized by a series of Hermite surface sheet of a merger. Because of limited information for the reliable reconstruction and too many geometric assumptions, these methods can only reconstruct a rough skeleton model with low accuracy. Since the beginning of the 21st century, a new 3D reconstruction method based on bi-planar was proposed using a 3D generic model as a reference model. These studies extracted the features from 2D images, and then take adjustment and deformation operation to the 3D reference model based on the 2D-3D image registration technology, where 2D is the traditional X-ray film or DR, 3D is the CT or MRI volume data. The reference model provides general information of 3D object to be rebuilt so that it can achieve more accurate visualization of 3D model. For example, the Paris scholars [4] in Biomecanique Laboratory made use of the

positive and lateral X-ray films of the femurs to match the outline of the 2D images with that of the 3D reference model. They adopted the direct linear transformation (DLT, Direct Linear Transformation) algorithm to complete the 3D reconstruction of the 3D stereo correspondence points, non-stereo correspondence contour (NSCC) algorithm to complete the reconstruction of 3D non-stereo corresponding point, and the 3D interpolation and elastic deformation to adjust the reference model. Zhang [5] reconstructed the 3D corresponding points on the femur crosssection, and then used linear interpolation method to recover the 3D pose of femoral shaft. This method can be applied to other bones' reconstruction such as the ribs [6] and the spine [7]. Zheng et al [8] extracted the 2D projection image feature point pairs from the DR images, and established the 3D point distribution model (PDM) by 2D-3D registration method. They employed ray-tracing algorithm to produce the sets of 3D feature points between the contour of the 3D model projected and the edges of the image extracted, and finally obtained a 3D model of the individual patient with the 3D-3D registration method. Gamage et al [9] in the university of Auckland applied 2D-3D non-rigid registration method to 3D reconstruction for the long bones of the human body successfully. By using a new similarity measure for 3D pose estimation, which was a weighted sum of four kinds of feature measure, their method can effectively handle the noise, outliers, deformation, and blocked situations in a registration point set. The thin-plate spline interpolation (TPS) method was adopted for individual deformation of 3D model after pose estimation, and the accuracy was close to 3D reconstruction of CT. Matthews etc [10] also proposed a method to synthesize 3D femur model based on the combination of three techniques, namely DR images, CT tomography database and 3D fluoroscope scanning with the following three steps: 1) Determine the scope of femoral shaft fracture through the analysis of 3D fracture DR; 2) Select a reference mode closest to the patient's femur from the generic model library through a 2D-3D registration, where the femur proximal and distal bone segments of the reference model were taken as the approximations of uninjured femoral extremity, and the shaft of fracture femurs can be reconstructed directly by 3D fluoroscope scan device; 3) Combine each parts of the model together. In this way, an approximate 3D patient-specific femur model can be established.

This paper presents a 3D reconstruction of femurs method based on a 2D-3D non-rigid registration algorithm which is named as Coherent Point Drift (CPD) [11]. This method realizes the 3D reconstruction of patient-specific femur in preoperative with bi-planar DR and CT-scan data. The rest of this paper is organized as follows. After reviewing previous work in Section 1, we describe the Coherent Point Drift algorithm in Section 2 and 3D reconstruction methoed based on 2D/3D non-rigid registration in Section 3, and whole implementation flow is also discussed in this section. Section 4 presents experimental results with detailed analysis. Conclusions are drawn in the final Section 5 of this paper.

2. Coherent Point Drift Algorithm (CPD)

The 2D image registration is the core technology of 3D reconstruction. This paper adopted CPD algorithm to realize the non-rigid registration between the edges of digital radiographs and projected contours from 3D femoral generic model. CPD is a kind of non-rigid registration algorithm which is based on the probability of point sets. This algorithm is regarded as maximum likelihood estimation problem which is based on the velocity field movement consistency constraints and expressive movement consistent constraint by the variational method to regularize maximum likelihood estimation. We consider the alignment of two point sets as a probability density estimation problem, where one point set represents the Gaussian mixture model (GMM) centroids, and the other one represents the data points. At the optimum, two point sets become aligned and the correspondence is obtained using the maximum of the GMM posterior probability for a given data point. Core to this method is to force GMM centroids to move coherently as a group to preserve the topological structure of the point sets. This method not only estimates the complex nonlinear non-rigid transformation, but also has strong robustness under the conditions of noise and spill point.

We consider the points in $Y = (y_1, \dots, y_M)^T$ ($M \times D$ matrix) as the GMM centroids, and the points in $X = (x_1, \dots, x_N)^T$ ($N \times D$ matrix) as the data points generated by the GMM. D is the dimension of the point sets, and N, M is the number of points in the point sets. That Y_0 defines as the kernel of the initial position means the kernel of the current position is defined as

 $Y = v(Y_0) + Y_0$. The GMM probability density function is $p(x) = \sum_{m=1}^{M} \frac{1}{m} p(x \mid m)$, where $x \mid m \sim N(y_m, \sigma^2 I_D)$. Using Bayesian theory, we can obtain the *Y* by maximizing posterior probability or minimizing the energy function:

$$E(W) = -\sum_{n=1}^{N} \log \sum_{m=1}^{M} e^{-\frac{1}{2} \left\| \frac{x_n - y_{0m} - \sum_{k=1}^{M} w_k G(y_{0k} - y_{0m})}{\sigma} \right\|^2} + \frac{\lambda}{2} tr(W^T G W)$$
(1)

where $G_{M \times M}$ is symmetric Gram square matrix, element $g_{ij} = e^{-\frac{1}{2} \|\frac{y_0 - y_{0j}}{\beta}\|^2}$, $W_{M \times D} = (w_1, \dots, w_M)^T$ is Gaussian kernel matrix. λ is constant.

2.1. Optimization

We can acquire the upper bound of Equation (1) (E-step) by EM algorithm (Expectation Maximization) of the Gaussian mixture model clustering:

$$Q(W) = \sum_{n=1}^{N} \sum_{m=1}^{M} P^{old}(m \mid x_n) \cdot \frac{\|x_n - y_{0m} - G(m, R)W\|^2}{2\sigma^2} + \frac{\lambda}{2} tr(W^T G W)$$
(2)

where P^{old} denotes the posterior probabilities of GMM components calculated using the previous parameter values, G(m, R) denotes m-th row of G. Minimize Q is equivalent to minimize Equation (1) of the energy function. We can get the Equation (3) (M-step) by derivation the Equation (2) on W.

$$\frac{\partial Q}{\partial W} = \frac{1}{\sigma^2} G(diag(P1))(Y_0 + GW) - PX) + \lambda GW = 0$$
(3)

where *P* is a posterior probability, $p_{mn} = e^{-\frac{1}{2}\left\|\frac{y_m^{dm} - x_n}{\sigma}\right\|^2} / \sum_{m=1}^{M} e^{-\frac{1}{2}\left\|\frac{y_m^{dm} - x_n}{\sigma}\right\|^2}$, Diag(R) indicates a diagonal matrix. 1 denotes a column vector of *P*. The linear system of equation is as followed by multiplied $\sigma^2 G^{-1}$ on both sides of Equation (3):

$$(diag(P1))G + \lambda \sigma^2 I)W = PX - diag(P1)Y_0$$
(4)

W is solved from the Equation (4) is M-step of EM algorithm. E-step need to calculate posteriori probability matrix P.

2.2. Noise Robustness

Considering the overflow points in GMM, Equation (4) is represented as Equation (5) through changing the posteriori probability matrix.

$$P_{mn} = e^{-\frac{1}{2} \frac{\|y_{m}^{out} - x_{n}\|^{2}}{\sigma} / (\frac{2\pi\sigma^{2}}{\alpha} + \sum_{m=1}^{M} e^{-\frac{1}{2} \frac{\|y_{m}^{out} - x_{n}\|^{2}}{\sigma} ||^{2}})$$
(5)

where $\alpha = \frac{(1 - outlier) \cdot N}{outlier \cdot M}$, $outlier \in [0,1]$ represents the noisy probability.

The CPD algorithm flow is as followed:

- 1. Initialize parameters λ, β, σ .
- 2. Construct G matrix, initialize $Y = Y_0$.
- 3. Deterministic Annealing $\sigma = \alpha \sigma$ Until convergence.

E-step: Calculate P;

M-step: solve W from Eq.(4);

Profile $Y = Y_0 + GW$;

where α represents the annealing rate, σ represents the temperature in the deterministic annealing algorithm, λ represents the weight value, β represents the intensity of the mutual influence between the two points.

3. 3D Reconstruction Method Based on 2D/3D Non-rigid Registration

Figure 1 illustrates the proposed framework describing a 3D reconstruction method that registers the generic 3D model of a femur to bi-planar DR of different patients. Digital radiographs are initially processed to extract the edges from the femoral boundary. CPD algorithm that is a non-rigid registration method is then performed between the edge points identified in DR and the projected contour points of the 3D general model acquired by CT-scans. The identified point correspondences will be obtained to create a 2D planar translational field in both frontal and lateral viewpoints. This translational field will identify the 3D pose estimation and deformations required by the 3D anatomical model in the equivalent viewpoints. Finally, a full 3D translational field will be created for 3D reconstruction with updating parameters.



Figure 1. The Main Process of the Proposed Framework of 3D Femoral Model Reconstruction

3.1. DR Imaging System Calibration and Feature Extraction Method 3.1.1. Calibration of DR imaging System Based on Pinhole Camera Model

Because the accuracy of 3D reconstruction was determined by the calibration accuracy of DR imaging system, the imaging system calibration must be implemented before 2D-3D registration for clarifying the relation between 2D image and 3D space. In this paper, the DR imaging system was described through a pinhole camera model [12] as shown in Figure 2. O_c is the radiographic source, P is a 3D object point, P' is the projection point of P on the image plane. The relationship between 3D point $P(X_w, Y_w, Z_w)$ and its 2D projection point P'(u, v) could be described in Equation (6):

$$Z_{c}\begin{bmatrix} u\\v\\1\end{bmatrix} = A_{1}\begin{bmatrix} R & T\\0^{T} & 1\end{bmatrix}\begin{bmatrix} X_{w}\\Y_{w}\\1\end{bmatrix} = A_{1}A_{2}\begin{bmatrix} X_{w}\\Y_{w}\\Z_{w}\\1\end{bmatrix}$$
(6)

 A_1 is the intrinsic parameters matrix, which describes the camera geometry and optical properties parameters, including the principal point O_c , the focal length f and so on; A_2 is external parameters matrix, which describes the position and posture of the camera in the world coordinate system, including the rotation matrix R and translation vector T.

As the traditional calibration board of chessboard type is no longer applicable for the Xray environment, a new PCB calibration board with a copper mesh is designed in this section and its DR images are acquired accordingly [13]. Then, the intrinsic and external parameters are computed by extracting the angular points on the DR of the calibration board. The whole proposed procedure consists of a closed-form solution, followed by a nonlinear refinement based on the maximum likelihood criterion.



Figure 2. The X-ray Camera Model

3.1.2. 3D Generic Model Reconstruction and DRRs Production

3D generic model reconstruction was the basis for 2D-3D registration about 3D reconstruction of femoral model. In this part, one of the femurs is chosen from the CT database (Figure 3) as the prior knowledge (generic model) for the patient-specific anatomical model. The 3D generic surface model used in our testing is segmented and reconstructed from the CT volume data based on Marching Cube (MC) algorithm [14]. Then, the 3D generic model is projected onto the image plane to obtain the projection images. In this paper, the DRRs (digitally reconstructed radiographs) [15] of the generic femur are acquired and the 2D projected contours are extracted from frontal and lateral viewpoints respectively. The relationship between 3D femoral generic model based on CT volume data and its bi-planar DRRs are shown in Figure 4. The DRRs of model are adopted the ray-casting algorithm and use the obtained parameters by calibration parameters which are obtained in Section 3.1.1 to set the position of virtual X-ray source.



Figure 3. Femoral Specimens



Figure 4. Generic Femur Model and its DRRs from Frontal and Lateral Views

3.1.3. Feature Extraction Using Live-wire Algorithm

The interactive live-wire algorithm is a method which segments images by computing the minimum cost between two arbitrary given points. The interactive movement of the given point using the mouse cursor causes the boundary to behave like a live-wire. By following the optimal path pointers, it adapts to the new minimum cost path from the free point back to the seed point. By constraining the seed point and free points to lie near a given edge, the user is able to interactively adjust the live-wire boundary around the object of interest. (1) Cost Formula

To extract the boundary from the image, the method utilize image feature to construct functions. Since a minimum cost path should be corresponded to an image component boundary, the stronger edge features should be lower local costs. Proposed l(p,q) is the cost from pixel p to its neighboring pixel q.

$$l(p,q) = w_G \times f_G(q) + w_Z \times f_Z(q) + w_D \times f_D(q)$$
(7)

(2) The optimal path selection

Dijkstra algorithms [16] can deal with the problem of path chosen. Firstly, we construct the graph tree which every two nodes have one and only one path. Then, a 'free' node is created by dragging mouse. Finally the computer achieve the minimum cost path, then link the 'free' node and the seed.

3.2. 3D Pose and Shape Reconstruction Using 2D/3D Non-rigid Registration

Based on the preparation, the contour point set of generic femoral DRR and patient femoral DR, which are considered as floating point set and reference point set respectively, are obtained. The core problem of 3D reconstruction based on 2D-3D registration is to obtain the essential spatial parameter of degree of freedom.

Firstly, the CPD algorithm is used to obtain the 2D registration parameters by image registration of femoral at frontal view and lateral view. Nine parameters can be obtained such as the scale factors belonging three axis (s_x, s_y, s_z) , the angles of rotation revolving around three axis (α, β, γ) and the translation of three axis (t_x, t_y, t_z) .

Secondly, the 2D registration parameters can be transferred to 3D registration parameters matrix by augmenting matrix based on the 2D-3D projection relation of the pinhole camera model as Figure 2 is shown. The 3D scale matrix $S_{3\times3}$, 3D rotation matrix $R_{3\times3}$ and 3D translation vector $T_{3\times1}$ can be constructed by nine parameters which are obtained before. Then the 3D pose matrix *G* can be obtained by using Equation (8).

$$G = \begin{bmatrix} S_{3\times3}R_{3\times3} & T_{3\times1} \\ 0 & 1 \end{bmatrix}$$
(8)

where $S_{3\times3} = diag(s_x, s_y, s_z)$ is the 3D scale matrix, $T_{3\times1} = (t_x, t_y, t_z)^T$ is the 3D translation vector, $R_{3\times3}$ is the 3D rotation matrix which can be resolved into three rotation matrix component: R_x , R_y and R_z . $R_{3\times3}$ can be written as Equation (9).

$$R_{3\times3} = R_z R_y R_x = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0\\ \sin \gamma & \cos \gamma & 0\\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \beta & 0 & \sin \beta\\ 0 & 1 & 0\\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0\\ 0 & \cos \alpha & -\sin \alpha\\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$
(9)
$$\begin{bmatrix} \cos \beta \cos \gamma & \sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma & \cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma\\ \cos \beta \sin \gamma & \cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma & \cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma\\ -\sin \beta & \sin \alpha \cos \beta & \cos \alpha \cos \beta \end{bmatrix}$$

At last, the 3D pose matrix G will be used on the 3D model of generic femur by Equation (10).

$$\begin{bmatrix} X_{w} \\ Y_{w} \\ Z_{w} \\ 1 \end{bmatrix} = G \begin{bmatrix} X_{w} \\ Y_{w} \\ Z_{w} \\ 1 \end{bmatrix} = \begin{bmatrix} S_{3\times3}R_{3\times3} & T_{3\times1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_{w} \\ Y_{w} \\ Z_{w} \\ 1 \end{bmatrix}$$
(10)

where, (X_w, Y_w, Z_w) is the coordinate of the 3D model of generic femur, (X_w, Y_w, Z_w) is the coordinate of the 3D model of patient femur after registration.

By now the 3D reconstruction of the patient-specific femur has been finished. The preoperative 3D visualization of patient-specific femur based on bi-planar DR is achieved. By the process mentioned above, it can be understand that the non-rigid 2D-3D registration has several advantages: it considered both the similarities and differences between the generic femur and patient-specific femur; it can estimate the rotation angle and translational dislocation of the patient-specific femur; meanwhile, it can obtain the femoral model which is as similar as possible to the real pose by scale deformation for 3D femoral model.

4. Results and Discussion

In this paper, the hardware and software of implementation environment for 3D reconstruction of patient-specific femur is as followed. Operation system: Windows XP professional 32-bit SP3; Hardware conFigureuration: Inter Pentium Dual-core E5800 CPU, Frequency 3.20GHz, RAM 2G, Inter G41 Express Chipset Graphics card; Software environment: MATLAB R2011b, Adobe Photoshop CS5, visualization Tool Kit (VTK). To validate the 3D reconstruction capabilities of the proposed framework, eight CT scanned cadaveric femurs (four pairs) which are shown in Figure 3 were utilized in a series of tests: three pairs among them were chosen as the femur of the patients and the associated X-ray images were obtained in the frontal and lateral viewpoints; the other one pair of CT scanned femurs were used as the generic model for the left side and right side respectively.

This paper has performed the 3D reconstruction of the patient-specific femur based on the affine CPD algorithm, and compared with the results of iterative closest point algorithm (ICP) [17] registration (a classic points set registration method) and rigid CPD registration. Figure 5 shows the pose comparison before and after registration between patient-specific femoral contour and projected contour of generic femur at frontal viewpoint, where the thicker contour is the patient-specific femoral contour and thinner contour is projected contour of generic femur. The comparison of the 3D femoral model before and after registration are shown in Figure 6, where Figure 6 (a) is a generic femoral model, and Figure 6 (b) is the 3D reconstruction model of the patient-specific femoral model.





Figure 5. Comparison of the 2D Femoral Contours Before and After Registration

The key accuracy measurements used in this paper are the rotational errors, the translational average errors and successful registrations rate. These three indexes are evaluated the results of 3D reconstruction based on 2D-3D registration. We have achieved the 3D reconstruction experiments using ICP, rigid CPD and affine CPD registration algorithms, respectively. Table 1 shows 2D-3D registration experiment results.

As is vividly indicated by experimental results, the rotational average error and translational average error of 3D reconstruction based on ICP are less than 3° and 4mm. When the condition is restricted that the maximum translational error is less than 4mm or maximum rotational error does not exceed 3°, successful registration rate of ICP is about 60%; the maximum average rotational error of 3D reconstruction based on rigid CPD is 2.11°, and the maximum average translational error is 2.32mm, and the successful registration rate is as low as 90% with the tolerance limits; experiment results also indicate that the proposed 3D reconstruction method which is based on affine CPD algorithm possesses a higher precision and better robustness than ICP and rigid CPD registration with its average rotation and translation errors smaller than 2°, 2mm, respectively, based on a success rate more than 98%.

The factors of impacting femur 2D-3D registration experiment accuracy and robustness can be summarized as the following two aspects:

The first aspect is the degree of morphological differences between the common femurs and the patient's femurs. If there is a large morphological difference between the two femurs, extracted femur 2D contour data is not accurate enough, and the part contour feature is missing, it may lead to method failure or large error. For example, the morphology of the proximal femur is more complex than the distal femur, and the registration of the proximal femur is more difficult than the distal femur. Therefore, the proximal femur registration error is higher than the distal femur registration error.



a) Before Registration

b) After Registration

Figure 6. Comparison of the 3D Femoral Model Before and After Registration

The second aspect is the applicability of the transformation model with a registration method. First, the ICP and rigid CPD methods rely on the rigid transformation of a transformation model, thus they can only restore the three-dimensional rotation and translation attitude. Second, ICP and rigid CDP were not taken into account of the morphological differences between patients femur and common femur. Thus, they are not fully applicable to the non-rigid registration occasions between deformation registration object, and the experimental accuracy of the results and the robustness of the method are relatively low. In contrast, an affine CPD algorithm uses affine transformation as registration transformation model to increase the mirror and proportional deformation transformation and amend the morphological differences between the common femur and patient's femur. Therefore, affine CPD can recover the patient's femur more accurately, and it is significantly better than ICP and rigid CDP in terms of registration accuracy or robustness.

Table 1. Registration Experiment Results						
Evaluation index	ICP		Rigid CPD		Affine CPD	
	proximal	distal	proximal	Distal	proximal	distal
Rotation error/°	2.15	1.37	2.11	1.38	1.59	1.71
Translational error/mm	3.25	2.80	2.32	1.67	1.92	1.49
Successful registration/%	46.15%	61.54%	90.38%	100%	98.08%	100%

Table 1. Registration Experiment Results

5. Conclusion

This paper proposes a valuable method for the 3D reconstruction of femurs in bi-planar digital radiographs. The results indicate that the 3D reconstruction framework we proposed is able to identify the position and orientation of femur in a 3D environment from 2D orthogonal views with acceptable accuracies. Moreover, the proposed method has the potential to be further expanded to develop a system suitable for assisting orthopaedic surgeons in their pre-operative planning.

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