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# An Improved Algorithm under Error Correlation in Distributed Data Fusion

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## Abstract

In distributed data fusion, the correlation between every local estimate makes an impact on the result of fusion. This paper introduces a scalar of correlation coefficient to present the correlation between local estimates, and estimate a covariance matrix in the limit of correlation. The improved algorithm put forward to use the form of Bar shalom-Campo algorithm and partly estimate the limit of correlation in order to guarantee the consistency of fusion results and effectively utilize the information of correlation. By the comparison of the simulation experiments, the fusion accuracy of the proposed algorithm is proved to be more effective than that of the Bar shalom-Campo algorithm.

Keywords: distributed data fusion, error correlation, consistent, Bar Shalom-Campo algorithm

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# 1. Introduction

Since 1980s, the rapid development of the sensor and computer technologies has promoted the research of the technology of information fusion greatly. In order to meet the need of fight in military field, the technology of MSDF (Multi-sensor Data Fusion [1, 2]) is born at the right moment [3]. In MSDF, every sensor will preprocess their processor and send the intermediate result to center node to fusing, which is also called track fusion [4]. While the processing method of local navigation estimates error correlation is the main axis of the development of track fusion theory all the time [5].

In track fusion, it is an important question that the information of each information node is probably correlative and the degree of correlation is difficult to be acquired accurately. Under this condition, how to do estimation fusion and make the fusion result consistent is a difficult problem.

The earliest simple convex combination algorithm ignored the error correlation [3]. Fixed cross-covariance combination algorithm takes the error correlation between local estimation into consideration [5, 6]. While effectively using prior information, optimal distributed fusion algorithms with and without feedback also have considered the error correlation of local estimates and global estimates but have ignored the error correlation between local estimates [7, 8]. Linear minimum variance unbiased estimate algorithm has considered not only the error correlation of local estimates and global estimate and global estimate, but also the error correlation between local estimates [9-11]. In actual project, because of the impact of communication limits and filter characteristics, the information of cross covariance even covariance cannot be sent from local tracks to fusion center. The general solution for this situation is to calculate approximately the cross covariance and the covariance, then reconstruct them in the fusion center [12-15].

CI algorithm takes the correlation into consideration. In order to guarantee the consistence of fusion results, CI algorithm has considered all correlation .It results in that that under a condition that some parts of correlative information can be obtained, the algorithm is too conservative to effectively use the information of correlation. Instead, Bar shalom–Campo algorithm needs to utilize the information of correlation and has to estimate the correlation very accurately. Otherwise the consistency of the fusion results could not be guaranteed [5]. Actually, it is very hard to estimate accurately the covariance matrix at the moment. But under some

conditions, we can partly estimate the limit of correlation. In this way, the correlation of estimate error of every sensor can be taken into consideration. But in order to calculate the cross-covariance matrix of estimate error of every sensor, a large amount of information is needed.

This paper introduces a scalar of correlation coefficient to present the correlation between local estimates, and estimates a covariance matrix in the limit of correlation. The improved algorithm put forward uses the form of Bar shalom-Campo algorithm and partly estimates the limit of correlation, thus guaranteeing the consistency of fusion results and effectively utilizing the information of correlation.

# 2. The Correlation of Data Fusion

There are two kinds of reasons which result in the correlation of local estimate error of every fusion node.

The error correlation is generated because of common process noise and relevant measurement noise between the local state estimates and common prior estimates.

When the fusion center has the ability of remembering and has a lot of channel to transmitting information from sensors to fusion centers, there exist correlations between local state (prior) estimate and global state (prior) estimate.

First ,given the measurement and estimate of multi-sensor come from the same target, the previous state estimate matrix and covariance matrix of sensor *i* and *j* are respectively  $^{m}$ 

shown as  $x_{0|0}$  and  $P_{0|0}^m$ , *m=i,j*. The dynamical equation of target is shown as follow:

$$x_{k+1} = \Phi_k x_k + \Gamma_k w_k \tag{1}$$

where the process noise Wk is white noise with zero-mean,  $Q_k$  stands for covariance matrix. The measure equation of two sensors is as follows:

$$z_{k}^{m} = H_{k}^{m} x_{k} + v_{k}^{m}, m = i, j$$
<sup>(2)</sup>

where measurement noise vkm is white noise sequence with zero-mean, the Rkm means the covariance ,and they are mutual independent.

When the time is k, state estimate of measurement information of sensor i can be expressed as follow:

$$\hat{x}_{k|k}^{i} = \Phi_{k-1}\hat{x}_{k-1|k-1}^{i} + K_{k}^{i}(z_{k}^{i} - H_{k}^{i}\Phi_{k-1}\hat{x}_{k-1|k-1}^{i})$$
(3)

where Kki means the gain matrix of Kalman filter, the corresponding estimate error is shown as this:

$$\begin{aligned} \tilde{x}_{k|k}^{i} &= x_{k} - \hat{x}_{k|k}^{i} \\ &= \varPhi_{k-1} x_{k-1} + \varGamma_{k-1} w_{k-1} - \varPhi_{k-1} \hat{x}_{k-1|k-1}^{i} \\ &- K_{k}^{i} [H_{k}^{i} (\varPhi_{k-1} x_{k-1} + \varGamma_{k-1} w_{k-1}) + v_{k}^{i} - H_{k}^{i} \varPhi_{k-1} \hat{x}_{k-1|k-1}^{i}] \\ &= (I - K_{k}^{i} H_{k}^{i}) \varPhi_{k-1} \tilde{x}_{k-1|k-1}^{i} + (I - K_{k}^{i} H_{k}^{i}) \varGamma_{k-1} w_{k-1} - K_{k}^{i} v_{k}^{i} \end{aligned}$$
(4)

So the cross-covariance matrix between local estimate errors of sensor *i* and *j* can be written as

$$P_{k|k}^{ij} = E[\tilde{x}_{k|k}^{i} \left( \tilde{x}_{k|k}^{j} \right)^{T}]$$
  
=  $P_{k|k}^{i} \left( P_{k|k-1}^{i} \right)^{-1} [\mathcal{\Phi}_{k-1} P_{k-1|k-1} \mathcal{\Phi}_{k-1}^{T} + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^{T}] (P_{k|k-1}^{j})^{-1} P_{k|k}^{j}$  (5)

From the equations mentioned above, because of the prior estimate  $P_{k-1ij}$ , process noise  $Q_{k-1}$ , measurement noise  $R_{k-1ij}$  and potential impact of correlation of local estimate error of every sensor at the initial time, the local estimate error of any tow sensors *i* and *j* is relevant, which should be taken into consideration when fusing data.

Given the real state of target is *x*, state estimate of target from the sensor is *x*, the estimated error covariance is *P*, the real error covariance is  $\vec{P} = E(\hat{x} - x)(\hat{x} - x)^T$ . The so-called consistency means  $P \ge P$  [3,12-18].

# 3. An Algorithm under Error Correlation in Distributed Data Fusion 3.1. The Introduce of the Algorithm

When both of the two tracks which join in fusion are sensor track or one of them is sensor track, another one is system track, and the fusion center will give the feedback to every sensor after fusing [4], namely

$$\begin{cases} \hat{x}_{k-1|k-1}^{m} = \hat{x}_{k-1|k-1} \\ P_{k-1|k-1}^{m} = P_{k-1|k-1}, m = i, j \end{cases}$$
(6)

At this time, the computational formula of cross-covariance between local estimate errors the sensors that equation (5) describes can be written as:

$$P_{k|k}^{ij} = E[\tilde{x}_{k|k}^{i} \left(\tilde{x}_{k|k}^{j}\right)^{T}]$$

$$= P_{k|k}^{i} \left(P_{k|k-1}^{i}\right)^{-1} [\mathcal{\Phi}_{k-1}P_{k-1|k-1}\mathcal{\Phi}_{k-1}^{T} + \Gamma_{k-1}Q_{k-1}\Gamma_{k-1}^{T}] (P_{k|k-1}^{j})^{-1}P_{k|k}^{j}$$

$$= P_{k}^{i} P_{k|k-1}^{i-1} P_{k|k}^{j}$$
(7)

In consideration of the correlation of local estimate error of the sensors (7) mentioned above, the corresponding fusion equation and error covariance matrix can be expressed respectively like this:

$$\begin{cases} \hat{x} = \hat{x}^{i} + (P^{i} - P^{ij})(P^{i} + P^{j} - P^{ij} - P^{ji})^{-1}(\hat{x}^{j} - \hat{x}^{i}) \\ P = P^{i} - (P^{i} - P^{ij})(P^{i} + P^{j} - P^{ij} - P^{ji})^{-1}(P^{i} - P^{ji}) \end{cases}$$
(8)

Cross-covariance matrix  $P_{ij}$  can be calculated by the equation (5).Actually, it is very hard to estimate accurately the covariance matrix at the moment. But under some conditions, we can partly estimate the limit of correlation. In this situation, this paper proposes that given the real error correlation coefficient between two local estimates is  $\xi$ ,  $a \le b, \xi \in [a, b]$ , and

there is a cross-covariance matrix estimate  $P_{ab}$  satisfying this condition:

$$a^{2}P^{j} \leq P^{ji} \left(P^{i}\right)^{-1} P^{ij} \leq b^{2}P^{j}, \quad a^{2}P^{j} \leq \hat{P}^{ji} \left(P^{i}\right)^{-1} \hat{P}^{ij} \leq b^{2}P^{j}$$
(9)

Under this condition, making full use of the information of correlation can improve the fusion accuracy.

Algorithm process:

(1) After acquiring the estimate of a, b and P, we can define the combination covariance matrix as follows:

$$P^* = \begin{bmatrix} [1+(b-a)]P^i & \hat{P}^{ij} \\ \hat{P}^{ji} & [1+(b-a)]P^j \end{bmatrix}$$
(10)

(2) Judging from the equation (8), the weighs of fusing can be made certain of to do fusion.

The method of estimating the limit of correlation coefficient can be referred to literature [19, 20]. The cross-covariance matrix can be estimated by the equation (5), and the estimated  $\hat{P}^{j}$  should satisfy this condition  $a^2 P^j \leq \hat{P}^{j'} (P^i)^{-1} \leq b^2 P^j$ .

# 3.2. Algorithm Demonstration

The demonstration of the algorithm consistency:

$$P^{*} - P \ge 0 \Leftrightarrow \begin{bmatrix} [1 + (b - a)]P^{i} & \hat{P}^{ij} \\ \hat{P}^{ji} & [1 + (b - a)]P^{i} \end{bmatrix} - \begin{bmatrix} P^{i} & P^{ij} \\ P^{ji} & P^{i} \end{bmatrix} \ge 0$$
$$\Leftrightarrow \begin{bmatrix} (b - a)P^{i} & \hat{P}^{ij} - P^{ij} \\ \hat{P}^{ji} - P^{ji} & (b - a)P^{j} \end{bmatrix} \ge 0$$
(11)

Given the real correlation coefficient is  $\xi$  , then

$$a \le \xi \le b \tag{12}$$

$$P^{ji} \left(P^{i}\right)^{-1} P^{ij} = \xi^{2} P^{j}$$
(13)

$$a^{2}P^{j} \leq P^{ji} \left(P^{i}\right)^{-1} P^{ij} \leq b^{2}P^{j}$$
(14)

s 
$$a^2 P^j \le P^{ji} \left(P^i\right)^{-1} P^{ij} \le b^2 P^j \Leftrightarrow \forall x, y$$
 (15)

where  $(P^{j})^{-1} = Q_{i}^{T}Q_{i}, (P^{j})^{-1} = Q_{j}^{T}Q_{j}$ . Seen from the three equations mentioned above:

$$(\hat{P}^{ij} - P^{ij})^T (P^i)^{-1} (\hat{P}^{ij} - P^{ij}) \le (b^2 - \xi^2) P^j \le (b^2 - a^2) P^j$$
(16)

Setting  $\chi$  as a arbitrary vector, and  $\chi^{T} = [Q_{i}x, Q_{i}y]$ 

$$\chi^{T} \begin{bmatrix} (b-a)P^{i} & \hat{P}^{ij} - P^{ij} \\ \hat{P}^{ji} - P^{ji} & (b-a)P^{j} \end{bmatrix} \chi$$
  
=  $(b-a)(|x|^{2} + |y|^{2}) + 2|x||y|x^{T}Q_{i}^{T}(\hat{P}^{ij} - P^{ij})Q_{i} y$   
 $\geq (b-a)(|x|^{2} + |y|^{2} - 2|x||y|)$   
=  $(b-a)(|x| - |y|)^{2}$   
 $\geq 0$  (17)

The consistency of the algorithm has been proved.

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#### 4. Simulation Experiment

For comparing the algorithm proposed by this paper with the traditional algorithm, we choose a simulation model as follows:

$$x_{k} = 0.5 x_{k-1} + w_{k-1}$$

$$z_{k}^{i} = 0.9 x_{k} + v_{k}^{i}$$

$$z_{k}^{j} = 0.85 x_{k} + v_{k}^{j}$$
(18)

Where  $E\{w_k\}=0$ ,  $E\{v_k\}=0$ ,  $E\{w_kw_j\}=0.01\delta_{ki}$ ,  $E\{v_kv_j\}=0.5\delta_{ki}$ ,  $x_0=0.1, P_0=2$ . When  $k \neq j$ ,

 $\delta_{kj}$  =1. With *(b-a)* changing from 0 to 1, the changing range of  $P_{ab}$  gets larger and different

estimate of  $P_{ab}$  has great impact on fusion results, so the error of fusion results gets larger and we set (b-a)=0.1. Using the state of equation (18) and the measurement equations of two sensors to generate simulation data, the improved algorithm proposed by this paper is simulated in track fusion.

Figure 1 shows the state estimate of two sensors, system state and state estimate of improved Bar shalom-Campo algorithm and Bar shalom-Campo algorithm.

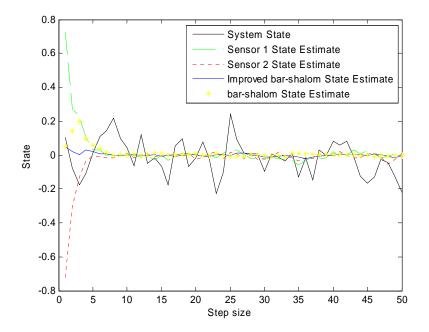


Figure 1. State estimate for improved bar-shalom and bar-shalom

Figure 2 describes the estimation variance curve of improved Bar shalom-Campo algorithm and Bar shalom-Campo algorithm, which proves that the fusion algorithm have a relative good predicting accuracy and low uncertainty.

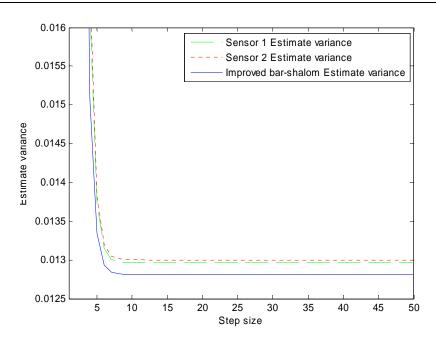


Figure 2. Estimate variance for improved bar-shalom and bar-shalom

Figures 3 and 4 describe the state estimate comparison of sensors and fusion state correlation when (b-a)=0.1. This example has proved that when the estimate accuracy is relative high, the accuracy of fusion results of the proposed fusion algorithm with measurement noise and two sensors is higher and more stable than Bar shalom-Campo algorithm.

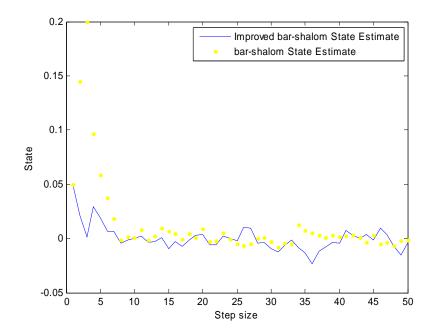


Figure 3. State estimate comparison for improved bar-shalom and bar-shalom

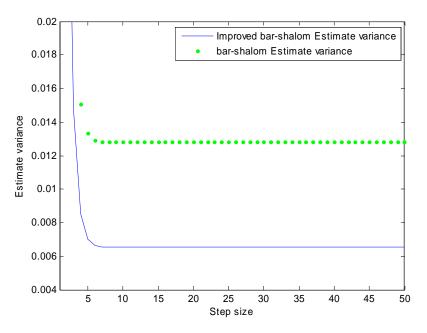


Figure 4. Estimate variance for improved bar-shalom and bar-shalom

## 5. Conclusion

In distributed data fusion, it is an important question that how to deal with the correlation between every local estimate [21-25]. This paper studied how to utilize the information of correlation under the condition that some parts of information of correlation. The paper proposed a scalar of correlation coefficient to present the correlation between local estimates, and estimate a covariance matrix in the limit of correlation. This algorithm uses the form of the improved Bar shalom-Campo algorithm. Experiments indicate that the calculated amount of our algorithm is relative small and the fusion accuracy is higher than the Bar shalom-Campo algorithm.

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