

The Application of Compressive Sensing on Spectra De-noising

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Abstract

Through the analyzing of limitations on wavelet threshold filter de-noising, this paper applies wavelet filter based on compressed sensing to reduce the signal noise of spectral signals, and compares the two methods through experiments. The results of experiments shown that the wavelet filter based on compressed sensing can effectively reduce the signal noise of spectral signal. The de-noising effect of the method is better than that of wavelet filter. The method provides a new approach for reducing the signal noise of spectral signals.

Key words: compressed sensing, sparse representation, spectral de-noising

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1. Introduction

Spectral measurement technology has been widely used in the field of atmospheric remote sensing monitoring technology and atmospheric chemistry research. Based on its advantages, it is quite suitable for real-time monitoring in large scale field. But in the process of acquiring near infrared spectrum, there are many noises were brought for the factors such as spectrum instrument itself, band range of ground spectrometer, resistance thermal noise of spectrum signal, random error, etc. Factors of ground environment and atmospheric (illumination conditions), measuring by airborne sensing instrument and space-borne sensing instrument can also change the spectral characteristics. Thus, there may be noise interference in the spectrum we have gotten [1]. So, the spectral signal which obtained from spectrometer should be de-noised to reduce the interference from external factor that disturb on the spectrogram.

There are many spectral de-noising method at present, i.e. wavelet threshold method [2], Gaussian filter [3] and wavelet packet domain de-noising method [4]. But those methods are always used with the condition that there is a large difference between signal frequency and noise frequency. However, current research still focus on signal de-noising which the frequency of noise is close to the frequency of signal itself. Such as Donoho put forward de-noising by using wavelet threshold [5], in this method a threshold value was given, the algorithm reserve the signals whose wavelet coefficient amplitude value is greater than the threshold value, otherwise wipe it off. In general, the substance of wavelet de-noising is that separate the useful information and noise with spectrum in frequency domain. But high frequency part also exists in spectrum. This kind of method lost many parts of useful information while removing noise.

The main idea of CS (Compression Sensing) is acquiring linear projection of signal in a given region, and then reconstructing the original signal by corresponding reconstruction algorithm [6]. It is pointed out in literature [7] that the wavelet coefficients of noiseless signal got by Mallat algorithm should be sparse, that is to say there is only a few large amplitude coefficient in the dimensions of the wavelet coefficient, but when noise exist in the signal, the sparseness of wavelet coefficients will be reduced greatly. Therefore, we can recover the sparseness of wavelet coefficients by using compression sensing to achieve the purpose of the signal de-noising. It can avoid threshold selection in wavelet threshold filtering which always difficult to choose by using this method.

In this paper we proposed spectral signal de-noising method based on compression sensing, which is contraposed the shortcomings of traditional wavelet de-noising method. It overcome the current problem of wavelet de-noising, and achieved better de-noising result. And the base of the signal decomposition in this paper is learning dictionary. While in the most other related researches, the dictionary is based on a certain redundant dictionary such as Gabor or Curvelet etc. The algorithm of learning dictionary is designed to find the optimal dictionary. Based on the sparse representation of this dictionary, it can minimum mean square error of approximation for samples. Compared with fixed dictionary, there are many advantages by using learning dictionary, such as more flexible and avoid many limitations. On the other hand, it can save much signal processing time by using learning dictionary.

2. Sparse Representation

In 1993, Mallet and Zhang first proposed that through the decomposition of signal in the over-complete dictionary, the base of signal (which was called dictionary) can be represented flexible in the way of selecting bases on the characteristics of the signal itself. This concise expression which comes up from the signal decomposition is called sparse represents [8]. Sparse representation includes two parts that are the process of dictionary selected and the process of sparse coding. The selection of the dictionary can be divided in two methods, the way based on the over-complete dictionary and the way based on learning dictionary. In over-complete dictionary based method, an atomic library is given to the signal (the atoms in the atomic library can either be orthogonal or non-orthogonal), through this way to solve the component expression of the signal in the atomic library. Dictionary learning means to learn a basic matrix D from training pattern, making each sample can be better represented with multiplied by a dictionary D and the coefficient of the vector [9]. Sparse coding means the process of solving sparse coefficient vector from a given base matrix D . It can be presented as:

$$\min \|\alpha\|_0 \text{ subject to } x = D\alpha \quad (1)$$

Here $\|\cdot\|_0$ represents zero norm, that is the number of non-zero elements.

But Donoho pointed, problem of solving the smallest L0 norm is essentially a NP-hard (Nondeterministic Polynomial-time hard) problem. The issue usually needs to be converted. The researchers made a series of obtained suboptimal solutions, which including algorithm of minimum L0 norm, matching Pursuit algorithm, the iterative threshold method and smallest full variational method [6]. It is proved theoretically that in the sparse coding L1 norm and L0 norm are equivalent when under some certain conditions [10] [11].

3. Compressed Sensing

The specific meaning of the sparse representation is to use as few elements as possible to represent the information. The core idea of compressed sensing is to obtain sparse representation of the signal and reconstruct the original signal through reconstruction algorithm [12] [13] [14].

For an unknown signal, if it is K-sparse or it becomes K-sparse by a known transform, it does not need as much coefficient as the Nyquist principle demanded (which demand the sample frequency is at least the twice as the highest frequency of the original signal) to reconstruct original signal accurately in the linear transformation. This is the basic idea of the CS theory. Let $\alpha(n)$ be the N-dimensional digital signal which obtained from the conventional sampling, and $y(m)$ is M-dimensional sampled signal which obtained by compressing sensing theory, wherein $M < N$. The relationship between y and α can be expressed as $y = \Phi\alpha$. Φ is the observation matrix or the measurement matrix which size is $M \times N$. This formula can be regarded as a linear projection of the original signal α at Φ . As the dimension of y (presented with M) is far less than the dimension of α (presented with N), there are infinitely many solutions according to the equation y to finding α . We can reconstruct the original signal through the way of solving the optimal solution of linear programming problems [15].

In the theory of compressed sensing, the most fundamental basis of signal recovering is spares characteristics at projection of original signal in a transform space. But the presence of

noise has destroyed the sparsity of the signal in the transform space. When using optimization method to restore the signal, if we treat noisy signal by using a single sparse constraint principle, the original signal cannot be reconstructed accurately. Nevertheless, compressed sensing theory can still be reconstructed effectively. The main difference is in the form and parameter settings in process of reconstruction of optimization objective function. Using different optimization objective functions, the signal reconstruction effects are different [15].

That is to mean, if there is a signal $y(y \in R^M)$ which length is M , the base vector for the signal is $\varphi_i(i = 1, 2, \dots, M)$. Make the transformation of:

$$y = \sum_{i=1}^M \alpha_i \varphi_i \quad \text{or} \quad y = \Phi \alpha \quad (2)$$

In this equation, α is the sparse representation of signal. Whether the signal has sparsity or approximate sparsity is the critical issues which decide success of using of the compressed sensing theory [16].

For signal de-noising, first we need to solve L1 or L2 norm of signals and get sparse representation, then use the transfer matrix Φ to reconstruct the signal. Then the signal de-noising is realized.

The problem of optimization in signal reconstruction process is very similar to the problem with optimization in signal sparse decomposition.

So some scholars looking for more effective ways of solving signal from sparse decomposition theory. The solutions which are commonly used are Basis Pursuit (BP), Matching Pursuit (MP) and Orthogonal Matching Pursuit (OMP) etc.

4. Spectral De-noising Model Based on Compressed Sensing

A. De-noising model

The problems of spectral noisy can be modeled as:

$$y = x + n \quad (3)$$

Here y is the observation noisy signal, x is the original signal and n is independent zero-mean additive white Gaussian noise. According to the theory of compressed sensing, it can be representing as:

$$y = x + n = \Phi \alpha \quad (4)$$

α is sparse representation of the signal α under transform Φ . If the spectrum is clean signal which means without any noise, we may solve the problem of:

$$\alpha = \arg \min \|\alpha\|_0 \quad s. \quad t. \quad \Phi \alpha = x \quad (5)$$

The clean spectrum signal has sparse representation. After adding noise, the sparse representation will be destroyed by the noise. To reconstruct the original signal, it can be achieved the noise removal by estimate the sparse representation of the clean signal. That is, when the signals join the additive white Gaussian noise, solving:

$$\alpha = \arg \min \|\alpha\|_0 \quad s. \quad t. \quad \|\Phi \alpha - y\|_2^2 \leq T \quad (6)$$

It can estimate a clean spectrum sparse representation by solving equation (6) and then resume the reconstruction of the signal, thereby noise is removed.

In order to make the reconstruct signal and observation signal maintain consistency, data fidelity constrains are introduced:

$$x = \arg \min_x \|x - y\|_2^2 \quad (7)$$

Taking into account the sparse prior, based on sparse representation discussed in the previous section, signals for each segment have:

$$D, \{\alpha_i\} = \operatorname{argmin} \|\alpha_i\|_1 \quad s.t. \quad \|\theta_i x - D\alpha_i\|_2^2 \leq \varepsilon_i \quad (8)$$

Where α_i denotes the representation coefficient for each segment, θ_i means extract paragraph i of signal, and ε_i is the error item.

Combined (7) and (8), a unified maximum a posteriori probability function is formed as

$$D, x, \{\alpha_i\} = \operatorname{argmin}_{D, x, \alpha_i} \|x - y\|_2^2 + \sum_{i=1}^N \lambda_i \|\alpha_i\|_1 \quad \text{subject to} \quad \|\theta_i x - D\alpha_i\|_2^2 \leq \varepsilon_i \quad (9)$$

Here λ_i is the regularization parameter.

B. Algorithm Description

The algorithm of this paper is divided into three main steps:

Step 1: Add the independent zero-mean additive white Gaussian noise to clean signal.

Step 2: Get sparse representation of the noisy spectrum by using Learning Dictionary which includes procedures are shown as follow:

- (1) Learn a dictionary from the observation noise signal as the optimal estimation of (9). Make the observation noise signal arbitrarily small section can better use a dictionary to sparse representation. Then we make the learned dictionary as the optimal estimate of the dictionary, fixed dictionary and then use the observation noise signal as the estimate of original signal.
- (2) Fix signal, segment sparse representation update x , obtain sparse coefficient. We rewrite (9) into

$$\{\alpha_i\} = \operatorname{argmin}_{\alpha_i} \sum_{i=1}^N \lambda_i \|\alpha_i\|_1 \quad \text{subject to} \quad \|\theta_i x - D\alpha_i\|_2^2 \leq \varepsilon_i \quad (10)$$

This is the sparse coding process and we can use ℓ_1 -norm solution to solve the problem and to get sparse coefficients α_i .

- (3) With the updated coefficients fixed, we use the steepest descent method iterative update signal x . First, sub-signal by overlap area take average fusion get z :

$$z = \operatorname{argmin}_z \sum_i \|\theta_i z - D\alpha_i\|_2^2 \quad (11)$$

Then we use the steepest descent method to solve the convex optimization problem update x .

$$x = \operatorname{argmin}_x \|x - y\|_2^2 + \gamma \|x - z\|_2^2 \quad (12)$$

Here γ is regularization parameter to control the influence of the two parts.

- (4) Repeat execution (3) (4) not jump out from the loop until some iterative conditions are satisfied.

Step 3: Use OMP (Orthogonal Matching Pursuit) algorithm to find the optimal solution of formula (1). In this step, the sparse spectrum signal is reconstructed with noise removed. [17]

5. Experimental simulation

Next, we simulate the performance of spectral de-noising based on compressed sensing, and compare with de-noising algorithm through the hard threshold wavelet filter. The experimental data are the results that obtained from Matlab platform. Because it is difficult to obtain pure spectral signal, we use simulated spectral signal in the experiment to evaluate the effect of various parameters. L—the length of the signal is 1024, as shown in Figure (1). We add additive white Gaussian noise at the signal-to-noise ratio at 20db to the original signal in the experiment to simulate measurement noise spectrum. Figure (2) is the spectral signal with additive white Gaussian noise. Figure (3) is the signal which is processed by hard threshold de-noising via wavelet transform algorithm. Figure (4) is shown the reconstruction signal based on the theory of compressed sensing in this paper.

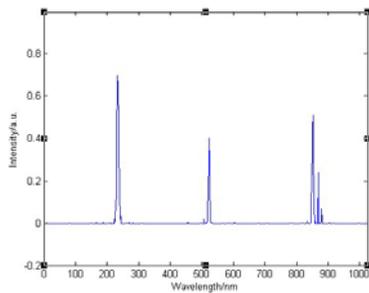


Figure 1. Original Spectral Signal

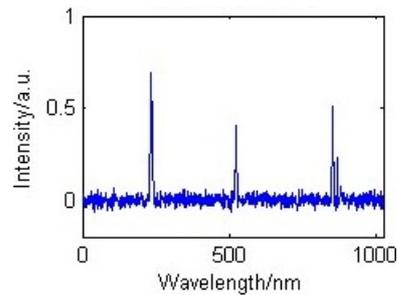


Figure 2 Original Spectral Signal with White Noise

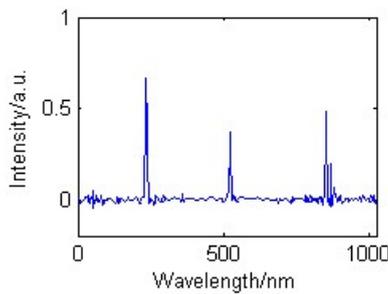


Figure 3. De-noising Spectral signal by hard threshold wavelet filter

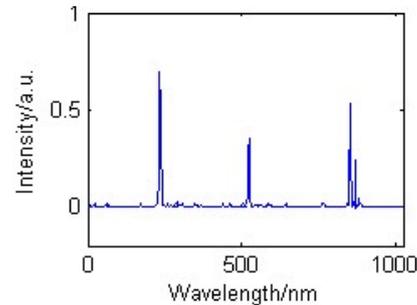


Figure 4. De-noising Spectral signal by CS

The evaluation parameters of de-noising effect involves signal-to-noise ratio (SNR), root mean squared error (RMSE), waveform similarity (NCC). Defined as follows:

$$SNR = 10 \times \log \left\{ \frac{\sum_{n=1}^N f_n^2}{\sum_{n=1}^N (f_n - s_n)^2} \right\} \tag{7}$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{n=1}^N (s_n - f_n)^2} \tag{8}$$

$$NCC = \left(\sum_{i=1}^N s_n f_n \right) / \sqrt{\left(\sum_{n=1}^N s_n^2 \right) \left(\sum_{n=1}^N f_n^2 \right)} \tag{9}$$

In the formula (7) to (9), $n = 1, 2, \dots, N$, s_n represent the original spectrum, f_n represent the spectrum after de-noising, N is the number of bands. SNR is on direct proportional to the de-noising effect; RMSE is on inversely proportional to the de-noising effect, and NCC is in the value interval of $[-1, 1]$ (-1 means waveform completely reverse after de-noising; 0 indicates that the two waveforms orthogonal, 1 shows they are completely the same) [18].

Table 1. Comparison Denoising Effects

Parameters	SNR	RMSE	NCC
Methods			
Hard Thresholding	32.0965	0.0127	0.9809
Compressed Sensing	33.7162	0.0117	0.9827

As can be seen in Table 1, SNR is better in the way based on compressed sensing. Compared with de-noising by the wavelet fixed threshold filtering, the de-noising spectrum is smoother by using compressed sensing.

6. Conclusion

This paper describes the theory of compressed sensing, and applied it into the de-noising of spectral signal. The comparative analysis of the experiment with the traditional wavelet threshold filtering method is shown in the paper. From the simulation results and other performance indicators, it is clear that the proposed method is better than wavelet threshold filtering method for spectral signal de-noising. We can get better evaluation parameters through compressed sensing.

Acknowledgments

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