

A Harmonic Analysis Method Based on Harris Window for Digital Substation

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Abstract

Accurate and efficient harmonics analysis is an important premise and basis for controlling the harmonics in power system. The sampling frequency of the grid voltage and current in digital substation normally must be a constant value as required by the standard of IEC61850, thus asynchronous sampling caused by the fluctuating grid frequency and this constant sampling frequency will result in spectral leakage and aliasing which would affect the accuracy of harmonic analysis. In this paper, a double-spectrum-line interpolation Fast Fourier Transform approach based on multi-term Harris window for grid frequency and harmonic measurement was proposed, and the simple and practical adjustment formulas for the frequency, amplitude and phase angle were derived by curve fitting. The correctness and effectiveness of the proposed method were validated by simulation and field test. The proposed method has higher accuracy in both the analysis of the harmonic amplitude and phase angle as compared with the conventional windowed method and, is more applicable to the harmonic analysis in the situation of digital substation.

Keywords: harmonic analysis, DFT, spectrum leakage, Harris window

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1. Introduction

The accurate and efficient detection of power system harmonic has great significance to the harmonic source location, harmonic controlling, safe and economic operation of power system. The present harmonic detection methods include zero-crossing technique [1], Taylor detection method [2], wavelet transform [3], least square method [4-5], Neural network technique [6-7] and Discrete Fourier Transform (DFT) [8-9]. Harmonic parameters could be calculated accurately by harmonic analysis based on DFT in conditions of synchronous sampling. While in actual power system, the system frequency is usually fluctuant continuously, and this will make it difficult to realize synchronous sampling even using the phase locked loop technique [10]. The standard IEC 61850 [11] for digital substation provides that the sampling rate of digital signal must be 20, 40, 80 or 200 times the power system fundamental frequency. Hence, the fluctuating grid frequency and this constant sampling frequency will result in asynchronous sampling, and this will cause the fence effect and spectrum leakage [12] unavoidably when using the DFT to analyze harmonic. At present, the windowed and interpolation FFT algorithm is usually adopted to restrain the fence effect and spectrum leakage and, the widely used window functions include Blackman window, Hanning window, Nuttall window [13-14] and so on. The side-lobe characteristics of these window functions, however, are not ideal and the effects of restraining spectrum leakage need to be improved.

In this paper, the Harris window which has better side-lobe suppressing characteristics is applied, and the side-lobe performance is improved further by increasing the order of Harris window function to inhibit the spectrum leakage more effectively. The simple and practical adjustment formulas of the frequency, amplitude and phase angle are derived by curve fitting. The digital simulation of complex power signals and the harmonic analysis of the actual signal are presented in this paper. The experimental results show that in the conditions of asynchronous sampling and non-integer-period truncation, the proposed algorithm can be used to calculate the fundamental frequency, harmonic amplitude and phase angle more accurately and effectively.

2. Spectrum Leakage due to No-synchronous Sampling

When the total sampling time is not an integer multiple of the actual signal cycle (i.e. asynchronous sampling), burrs will appear in the post-sampling signal due to the fact that DFT will repeat these sampling data to form a continuous cycle waveform, as illustrated in Figure 1(b). While in conditions of synchronous sampling, this phenomenon would not appear (as shown in Figure 1(a)). The apparent discontinuous burrs in Figure 1(b) will spread into the frequency spectrum, and result in the so called spectrum leakage phenomenon. Figure 2 is the frequency spectrum of the asynchronous sampling signal shown in Figure 1(b), the actual spectrum is presented by the dashed line. In conditions of asynchronous sampling, the frequency spectrum will be located at both sides of the real spectrum line, such as the line k_1 and k_2 in Figure 2. Windowed function can suppress the long-range frequency leakage (as shown in Figure 1(c)) by eliminating the significant discontinuous burrs, and interpolation can suppress the short range spectral leakage.

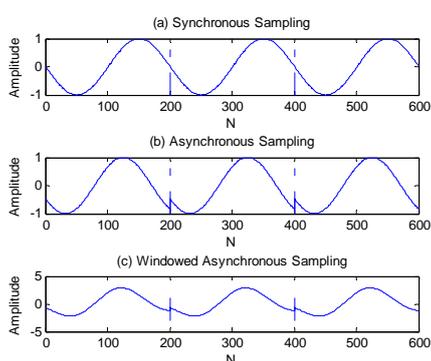


Figure 1. Asynchronous and Synchronous Sampling of the Same Signal

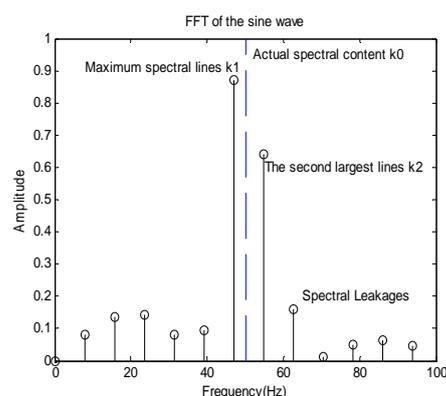


Figure 2. Frequency Spectrum of Asynchronous Sampling Signal

3. Multi-term Harris windows

3.1. Ideal Characteristics of Window Function

An ideal window function needs to satisfy the following conditions: 1) The width of the main side-lobe must be as smaller as possible to guarantee high frequency resolution; 2) The peak level of side-lobe must be as lower as possible to ensure good ability of noise detection and inhibiting; 3) The attenuation rate of side-lobe should be large enough. A window function that meets the above three criteria can eliminate the spectrum leakage and fence effect considerably. However, these criteria are usually contradictory and none of the currently used window functions can satisfy these three criteria simultaneously. So the key problem in designing the window function is to balance the main lobe with the side-lobe performance.

The characteristic and processing requirements of the signal are the key factors to be considered for the choice of window function. For example, if only the detection accuracy of the signal frequency is of the most concerned than that of the signal amplitude, such as the measurement of the natural frequency of a signal, a rectangular window is suitable due to the fact that the width of the main lobe is narrow to be distinguished easily. If the signal to be analyzed has narrow bandwidth and strong interference noise, the window function with small side-lobe should be chosen.

As for the analysis of voltage and current signals that contain various harmonics in power system caused by the accessing of numerous nonlinear loads, more attention should be paid to the side-lobe characteristics in choosing the window function.

3.2. Multi-term Harris Window

Harris window is a combination of cosine windows, and the general expression is as following:

$$w(n) = \sum_{m=0}^K (-1)^m b_m \cos\left(\frac{2\pi}{N} mn\right), \quad n = 0, 1, K, N-1 \tag{1}$$

$$b_m = \frac{\sinh(\pi\sqrt{\alpha^2 - m^2})}{\pi\sqrt{\alpha^2 - m^2}}, \quad m \leq \alpha - 1 \tag{2}$$

$$c = b_0 + 2\sum_{m=1}^{\alpha-1} b_m \tag{3}$$

$$a_0 = \frac{b_0}{c} \quad a_m = 2\frac{b_m}{c} \tag{4}$$

Coefficients b_m of Harris window function are shown in Table 1.

Figure 3 and Figure 4 show the time-domain and frequency characteristics of the 5-, 7- and 9-term Harris window functions respectively. It can be seen that the better side-lobe performance can be obtained by increasing the term of Harris window function.

Table 1. Coefficients of Harris Window

Terms	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7
4	0.3499	0.4850	0.1501	0.0150	/	/	/	/
5	0.3136	0.4661	0.1844	0.0339	0.0020	/	/	/
6	0.2867	0.4468	0.2070	0.0530	0.0063	0.0002	/	/
7	0.2657	0.4285	0.2217	0.0705	0.0125	0.0010	2.40E-05	/
8	0.2487	0.4117	0.2313	0.0857	0.0198	0.0026	1.52E-04	2.40E-06

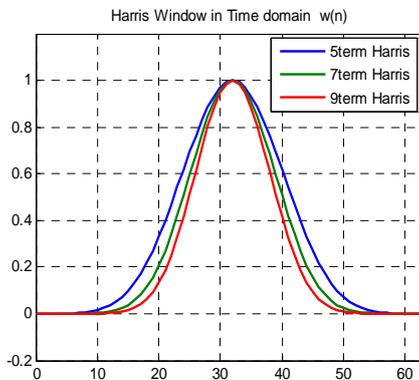


Figure 3. Harris Window in Time domain

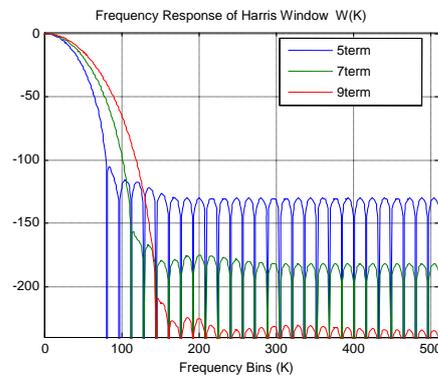


Figure 4. Frequency Characteristic of Harris Window

Table 2 gives the comparison of side-lobe performance between Harris window with other regular windows. It can be learned that Harris window functions are of good side-lobe characteristics.

Table 2. Comparison of the Side-lobe Performance of Multi Window Functions

window type	side lobe level (dB)	asymptotic decay (dB/oct)
Hanning	-32	18
Black-harris	-92	6
Nuttall	-82.6	30
3-term R-V	-46.8	30
4-term R-V	-61	42
5-term R-V	-82.6	54
6-term R-V	-88	66
7-term R-V	-101.1	78
8-term R-V	-114	90
9-term R-V	-126.8	114
6-term Harris	-130.9	24
7-term Harris	-156.7	18
8-term Harris	-182.7	12
9-term Harris	-209	6

3.3. Harmonic Analysis Method Based on Multi-term Harris Window

The specific steps of the harmonic analysis method based on multi-term Harris window proposed in this paper are: Firstly, the sampled discrete signal is windowed to obtain a data sequence with finite length; Secondly, DFT is used to analyze the data sequence to get the discrete spectrum of the windowed signal; Finally, the amplitudes of the two spectral lines that are the closest to the actual spectral line are interpolated.

Take the spectrum shown in Figure 2 as an example, the actual spectral line is k_0 , and the two closest lines are k_1 and k_2 , and the amplitudes of these two lines are y_1 and y_2 respectively. Two parameters are introduced as:

$$\beta = \frac{y_2 - y_1}{y_2 + y_1}, \quad \alpha = k_0 - k_1 - 0.5 \quad (5)$$

After getting the value of β , parameter α can be resolved by using the least square curve fitting method to perform polynomial approximation, and the frequency, amplitude and phase angle of the complex harmonic signal can be obtained ultimately.

The above simple and practical adjustment formulas for the frequency, amplitude and phase angle derived by polynomial approximation and double peaks line correction method are suitable to general window functions. For example, parameter α of the 9-term Harris window can be deduced as:

$$\alpha = 5.931 \times \beta + 0.729 \times \beta^3 + 0.327 \times \beta^5 \quad (6)$$

The adjustment formulas of the 6-term Harris window are:

$$f_0 = (\alpha + k_1 + 0.5) \Delta f \quad (7)$$

$$A = N^{-1} \times (y_1 + y_2) \times (10.39 + 6.26 \times \beta^2 + 19.99 \times \beta^4) \quad (8)$$

$$\theta = \arg[X(k_i \cdot \Delta f)] - \pi \cdot (\alpha - (-1)^i \cdot 0.5) \quad (i = 1, 2) \quad (9)$$

4. Simulation Verifications

The time-domain expression of the signal used in simulation is:

$$x(t) = \sum_{i=1}^{11} A_i \sin(2\pi f_i \times i \times t + \theta_i) \quad (10)$$

Where $f_1=50.5\text{Hz}$ is the fundamental frequency of the power system, and the sampling frequency is 10kHz as provided in IEC61850. The truncated data length N is 2048, i.e. 10.34 fundamental cycles, to guarantee asynchronous sampling. The amplitude A_i and phase θ_i of each harmonic are shown in Table 3.

Table 3. Parameters of the Harmonic Signals

harmonic	1	2	3	4	5	6	7	8	9	10	11
A_i (V)	220	4.4	10	3	6	2.1	3.2	1.9	2.3	0.8	1.1
θ_i (°)	0.05	39	60.5	12	-52.7	146	97	56	43.1	-19	4.1

The input signal $x(t)$ is processed respectively by 3-term Nuttall window, 5- and 6-term Rife-Vincent window, 6-, 7-, 8 and 9-term Harris window. The discrete spectrum can be obtained by FFT. Then the frequency, amplitude and phase angle of each harmonic can be calculated according to the double spectrum line interpolation formula of the window function.

The adopted adjustment formula for Nuttall window and R-V window are presented in Reference [13] and [14] respectively. The simulation results are shown in Table 4 and Table 5. Where D_{A_i} and D_{ϕ_i} are the relative deviation between the measured amplitude and phase angle of each harmonic with the actual value respectively; D_{f_0} is the relative deviation between the measured value of fundamental frequency with the actual value.

Table 4. Comparisons of Relative Errors in Calculating Amplitude and Frequency

Window type	D_{f_0}	D_{A_1}	D_{A_2}	D_{A_3}	D_{A_4}	D_{A_5}	D_{A_6}	D_{A_7}	D_{A_8}	D_{A_9}	$D_{A_{10}}$	$D_{A_{12}}$
Nuttall(III-4)	3.E-8	9.E-7	5.E-6	5.E-6	3.E-7	2.E-6	4.E-6	4.E-8	6.E-7	5.E-6	5.E-7	1.E-6
5-term R-V	3.E-10	1.E-7	7.E-8	4.E-6	3.E-8	1.E-0	2.E-6	5.E-8	7.E-8	8.E-7	4.E-8	8.E-9
6-term Harris	2.E-7	3.E-7	7.E-6	2.E-7	3.E-7	4.E-6	3.E-8	2.E-7	2.E-7	1.E-7	3.E-8	9.E-10
7-term Harris	2.E-11	2.E-7	2.E-7	5.E-6	1.E-7	2.E-7	2.E-6	2.E-8	1.E-7	1.E-6	5.E-8	1.E-8
8-term Harris	5.E-12	1.E-7	1.E-7	3.E-6	7.E-7	1.E-6	2.E-6	1.E-8	7.E-8	7.E-7	4.E-8	4.E-9
9-term Harris	4.E-13	8.E-8	9.E-8	1.E-6	5.E-8	8.E-8	5.E-7	9.E-9	5.E-8	3.E-7	3.E-8	2.E-9

Table 5. Comparisons of Relative Errors in Calculating Phase Angle

Window type	D_{ϕ_1}	D_{ϕ_2}	D_{ϕ_3}	D_{ϕ_4}	D_{ϕ_5}	D_{ϕ_6}	D_{ϕ_7}	D_{ϕ_8}	D_{ϕ_9}	$D_{\phi_{10}}$	$D_{\phi_{11}}$
Nuttall(III-4)	9.E-7	5.E-6	5.E-6	3.E-7	2.E-6	4.E-6	4.E-8	6.E-7	5.E-6	5.E-7	1.E-6
5-term R-V	9.E-6	8.E-5	5.E-7	9.E-7	2.E-6	-2.E-6	-6.E-7	8.E-6	-4.E-6	-5.E-6	3.E-5
6-term Harris	1.E-4	8.E-6	4.E-6	-1.E-6	-2.E-6	4.E-7	2.E-7	9.E-8	7.E-7	-3.E-6	1.E-5
7-term Harris	-9.E-7	-3.E-7	-6.E-8	-4.E-8	1.E-7	-1.E-7	-6.E-8	-7.E-8	-3.E-7	2.E-6	-7.E-6
8-term Harris	2.E-7	5.E-8	5.E-9	-5.E-9	-1.E-8	1.E-9	6.E-10	3.E-10	5.E-9	-3.E-8	9.E-8
9-term Harris	-2.E-8	-5.E-7	-3.E-9	-8.E-11	-5.E-9	-5.E-10	-1.E-10	-8.E-10	-9.E-10	4.E-9	-2.E-7

The curves of the measurement errors of amplitude and phase angle are shown in Figure 5 and Figure 6 respectively.

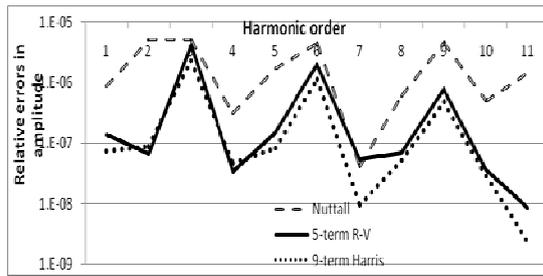


Figure 5. Comparisons of Relative Errors of Harmonic Amplitude

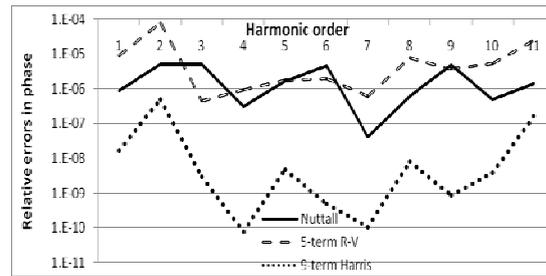


Figure 6. Comparisons of Relative Errors of Harmonic Phase Angle

As can be seen from the above results that the calculation error of the fundamental frequency using 9-term Harris window interpolation is 4×10^{-13} , and the calculation error of the amplitude and phase angle of fundamental component are 8×10^{-8} and 2×10^{-8} respectively. The calculation error of the amplitude and phase of 11 harmonic are 2×10^{-9} and 2×10^{-7} respectively. The accuracy of the harmonic analysis using Harris window is several orders of magnitude higher than that of the Nuttall window and R-V window interpolation method. In the conditions of the same term and degree of the correction formula, the algorithm proposed in this paper has a higher accuracy, which can achieve high accuracy in the analysis of complex harmonic signal.

5. Field test in digital substation

Harmonic analysis by using the proposed multi-term Harris window method is performed on the data collected by monitoring devices in a digital substation, to further verify the accuracy and practicability of the proposed method. The experimental procedure is shown in Figure 7.

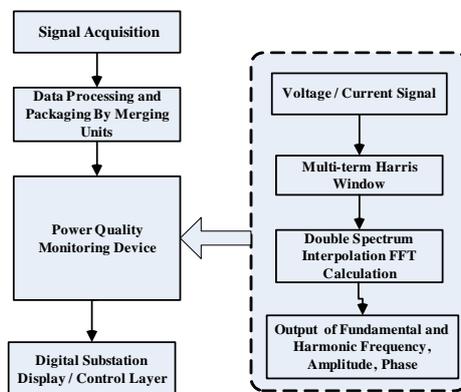


Figure 7. Flow Chart of Harmonic Analysis

The sampling frequency of the collected data is 10kHz and the total sampling time is 0.2s. The analyzed fundamental frequency of actual signal is 50.052Hz. The harmonic analysis results are shown in Table 7.

The relative error of fundamental amplitude and phase angle is 3×10^{-8} and 3×10^{-11} respectively, and the relative error of 49 harmonic amplitude and phase angle is 8×10^{-8} and 9×10^{-10} respectively. These results show that the proposed method can perform harmonic analysis with very high precision to the actual signals in digital substation.

Table 7. Harmonic Analysis Results of the Real Signal

Harmonic	Amplitude (V)	Relative errors of Amplitude	Phase (°)	Relative errors of Phase
1	10500	3.E-8	-42.2	3.E-11
2	5	9.E-8	13.9	2.E-5
3	6	4.E-8	-87	1.E-8
5	65	1.E-08	82.9	2.E-10
7	86	6.E-8	37	2.E-10
9	10	5.E-9	34.9	7.E-10
10	5	8.E-8	149	1.E-10
11	13	7.E-8	113	6.E-10
13	12	2.E-8	104	3.E-11
14	1	7.E-8	89	7.E-9
15	7	8.E-8	-60	3.E-9
16	1	2.E-6	107	3.E-7
17	5	4.E-8	-78.2	1.E-10
19	12	8.E-8	133	1.E-10
21	5	6.E-8	30	9.E-10
22	1	6.E-8	124	1.E-9
23	5	9.E-8	143	1.E-09
25	5	8.E-8	152	5.E-11
26	1	5.E-8	-162	2.E-09
27	2	9.E-8	62.3	8.E-09
28	1	9.E-7	138	-1.E-07
29	2	9.E-8	-82.6	-3.E-10
30	1	4.E-8	-12.1	1.E-08
31	6	9.E-8	-9.28	1.E-08
32	2	7.E-7	91.9	-1.E-07
33	3	1.E-7	133	-7.E-11
34	1	3.E-8	-54.4	-2.E-09
35	2	8.E-8	-27.3	2.E-8
37	2	1.E-7	2.16	5.E-9
38	1	2.E-8	-122	1.E-9
39	2	8.E-8	0.32	4.E-7
40	1	4.E-7	172	4.E-8
41	2	1.E-7	-84.1	4.E-10
43	1	7.E-8	-0.38	1.E-7
44	1	2.E-7	-139	4.E-8
45	1	1.E-7	53.8	1.E-9
46	1	8.E-9	-169	3.E-10
47	2	6.E-8	38.6	8.E-9
48	1	1.E-7	0.57	2.E-6
49	1	8.E-8	-56.3	-9.E-10

6. Conclusion

A method of frequency measurement and harmonic analysis based on multi-term Harris window is proposed in this paper. This method can perform fundamental frequency tracking and harmonic analysis with high precision even in the situations of asynchronous sampling, thus is more suitable to be applied to harmonic analysis in digital substation. The principle and realization of the double spectrum line interpolation algorithm based on multi-term Harris window is introduced in detail, and the simple practical interpolation formula is calculated by using the curving fitting. The results of simulation and practical application in digital substation show that the presented method is of less computation, higher accuracy and better practical value in engineering.

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