

# Comprehensive Evaluation of CNC Machine Tools Accuracy Based on AHP

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## Abstract

*In order to evaluate the accuracy of a numerical control machine tool comprehensively, the accuracy evaluation index system of a numerical control machine tool was established, and the priorities of accuracy indexes were determined by analytic hierarchy process method. Taking a five-axis numerical control machine tool for example, each accuracy index of it was studied by using fuzzy comprehensive evaluation, and a membership function was constructed by the intermediate form trapezoidal distribution. The evaluated result indicated that the accuracy of this numerical control machine tool was excellent. As a reference, the method presented in this paper could be used to evaluate the accuracy of other numerical control machine tools.*

**Keywords:** numerical control machine tool, accuracy evaluation, comprehensive evaluation, analytic hierarchy process

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## 1. Introduction

In recent years, CNC machine tools are widely used in the field of machinery manufacturing, and the machining accuracy is continuously improved. As a result, finishing technology has been developing rapidly. At the same time, the processing technology and machining accuracy put forward higher requirements of CNC machine tools in turn. Various problems of machine tool itself may lead to the substandard quality of the machined parts. Currently, most machine tool manufacturers are generally carried out quality inspection on parts after machining, and try to repair the problem if it is present. This method of low efficiency, high error rate, long cycle, may lead to long periods of downtime, so that the manufacturing cost will be increased greatly.

In view of the above problems, the ideal solution is to make regular checks on the accuracy of machine tools, and trace the source machine errors according to the changes. Through the compensation of each error source, the errors of machine tools will be eliminate or reduce, which enables the machine tool accuracy maintain in a required range. Therefore, it is urgent to establish a set of scientific, reasonable method to evaluate the precision of machine tools.

Currently, research on machine precision evaluation is not a lot. Chang-Tzuoh Wu has Studied on the geometry model for accuracy evaluation in numerically controlled milling machines [1]. Morimoto has developed a new method for evaluating the geometrical accuracy of a 5-axis machining center based on the measurement results of the machined work piece [2]. Wanhua Zhao has proposed a novel precision evaluation method for high-speed machine tool which takes the dynamic precision as index [3]. Fangyu Pan set up error model and accuracy calibration of 5-Axis machine tool by the theory of multi-body system [4]. The paper proposes a fuzzy comprehensive evaluation based on analytic hierarchy process (AHP) to evaluate the accuracy of CNC machine tools.

## 2. Fuzzy Comprehensive Evaluation

In practice, experts used to express the accuracy good or bad reviews on comprehensive evaluation about the accuracy of machine tools, or the ideal degree of an

accuracy index. These opinions expressed qualitatively machine precision level, but they were inevitably influenced by personal subjective factors and the prevailing environmental disturbances. Since the accuracy of CNC machine tools evaluation involves many indexes and is generally qualitative, quantitative methods can't be used directly comparative analysis with distinct blur characteristics, so the accuracy evaluation of CNC machine is a multi-level fuzzy comprehensive evaluation problem.

The concept of fuzzy sets was first used by American scholar Zadeh proposed in 1965 [5]. After several decades, fuzzy mathematics theory and fuzzy comprehensive evaluation method have made great progress, and have been widely applied in the field of mechanical engineering. Guangpeng Zhang proposed a dynamic characteristics of machine tool evaluation methods based on fuzzy mathematics theory [6]. Guiping Wang proposed the application of fuzzy extension of AHP evaluation method of CNC machine tools greenness [7]. Shihao Liu analyzed and evaluated the performance indicators of CNC machine tools using fuzzy comprehensive evaluation method [8]. Linlin Liu gave effective evaluation of the quality of course examination by using the fuzzy AHP comprehensive evaluation[9]. But there are almost none research of the accuracy evaluation of CNC machine using fuzzy comprehensive evaluation method. Therefore, this paper introduces fuzzy comprehensive evaluation method to conduct a comprehensive analysis and evaluation the accuracy of CNC machine tools.

### 2.1. Primary Fuzzy Comprehensive Evaluation

Fuzzy comprehensive evaluation method is a method based on fuzzy mathematics comprehensive evaluation method. The comprehensive evaluation changes the qualitative evaluation into quantitative evaluation based on fuzzy membership degree theory, which makes an overall assessment of objects restricted by many factors using fuzzy mathematics. Using fuzzy comprehensive evaluation method to evaluate the object, we need to construct grade fuzzy subset and determine the membership. Therefore the fuzzy indicators have been quantitative. Then we should use the fuzzy comprehensive transformation principle for each indicator. The most important step is to establish the fuzzy relationship  $R$  between the factor set  $U$  and the evaluation set  $V$ , and use mathematical statistics, expert valuation or AHP to determine the priority vector  $W$ . Finally we can get  $Y = WR$ , and complete the evaluation. This process is the primary fuzzy comprehensive evaluation [10].

### 2.2. Multi-level Fuzzy Comprehensive Evaluation

Since the accuracy of CNC machine tool is a multi-objective decision making problem, multi-level evaluation model must be applied. The principle of establish multi-level evaluation model is dividing the concerned factors into several levels by attributes. Comprehensive evaluation is conducted within each level preliminary, and then the comprehensive evaluation results of higher levels are completed. The steps of Multi-level fuzzy comprehensive evaluation is as follows [8]:

- a) Dividing factor set  $U=\{u_1, u_2, \dots, u_n\}$  into  $s$  subsets according to some property.

$$U_i = \{u_{i1}, u_{i2}, \dots, u_{ip}\}, i=1, 2, \dots, s \quad (1)$$

- b) For each sub-factor  $U_i$ , giving a single factor comprehensive evaluation. Let evaluation set  $V=\{v_1, v_2, \dots, v_m\}$ , and the priority vector of each factors in  $U_i$  is  $A_i$ :

$$A_i = (a_{i1}, a_{i2}, \dots, a_{in_i}) \quad (2)$$

Let  $R_i$  be the single factor matrix, then the judgement vector  $B_i$  is:

$$B_i = A_i \cdot R_i = (b_{i1}, b_{i2}, \dots, b_{im}), i=1, 2, \dots, s \quad (3)$$

- c) Conducting second level comprehensive evaluation for factor  $U$ . Taking each  $U_i$  as a factor of  $U$ , and  $B_i$  as its single factor evaluation vector, then membership matrix can be constructed as follows:

$$R = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_s \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & \dots & \vdots \\ b_{s1} & b_{s2} & \dots & b_{sm} \end{bmatrix} \quad (4)$$

Giving priorities distribution  $A$  according the importance of  $U_i$ ,

$$A = (a_1, a_2, \dots, a_s) \quad (5)$$

Then second level comprehensive evaluation vectors are:

$$B = A \cdot R = (b_1, b_2, \dots, b_m) \quad (6)$$

Second level fuzzy comprehensive evaluation model is shown in Figure 1.

If the factor set  $U_i$  ( $i = 1, 2, \dots, s$ ) still contains more factors, it can also be subdivided into third or fourth level. The factors that affect the accuracy of CNC machine tools are variable, so the paper proposes three-level fuzzy comprehensive evaluation model to evaluate.

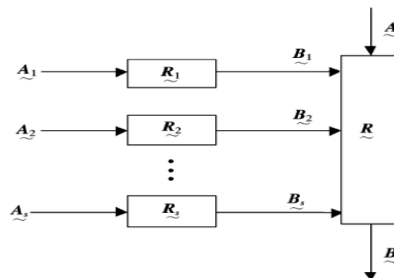


Figure 1. Second-level Fuzzy Comprehensive Evaluation Model

### 3. Accuracy Evaluation Model of CNC Machine Tools

#### 3.1. Establishing Accuracy Evaluation Index System of CNC Machine Tools

The factors affecting accuracy of CNC machine tools include geometric errors, position errors, thermal errors, processing errors, etc. In order to fully and accurately evaluate the accuracy of CNC machine tools, evaluation index system is established with AHP (Analytic Hierarchy Process) [11].

CNC accuracy evaluation index system is divided into three levels. The first level is the overall objective factors set  $U = \{U_1, U_2, U_3, U_4\}$ , as shown in Figure 2.

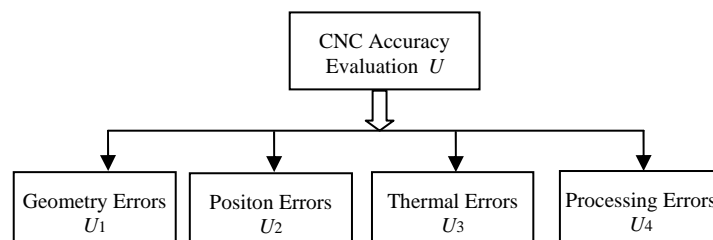


Figure 2. First Level of CNC Accuracy Evaluation Indexes System

The second level can be obtained by decomposition of the factors in first level. Geometric errors can be decomposed into straightness error, angle error, perpendicularity error and parallelism error. Position errors can be decomposed into linear axis position error and rotation axis position error. Thermal errors can be decomposed into X direction, Y direction, Z

direction thermal error and X-Axis, Y-Axis angle error. Processing errors can be decomposed into surface profile error, thickness deviation and surface roughness. Similarly, we can get the third level evaluation indexes. The evaluation indicators of geometry errors, position errors, thermal errors and processing error sare shown in Figure 3~5.

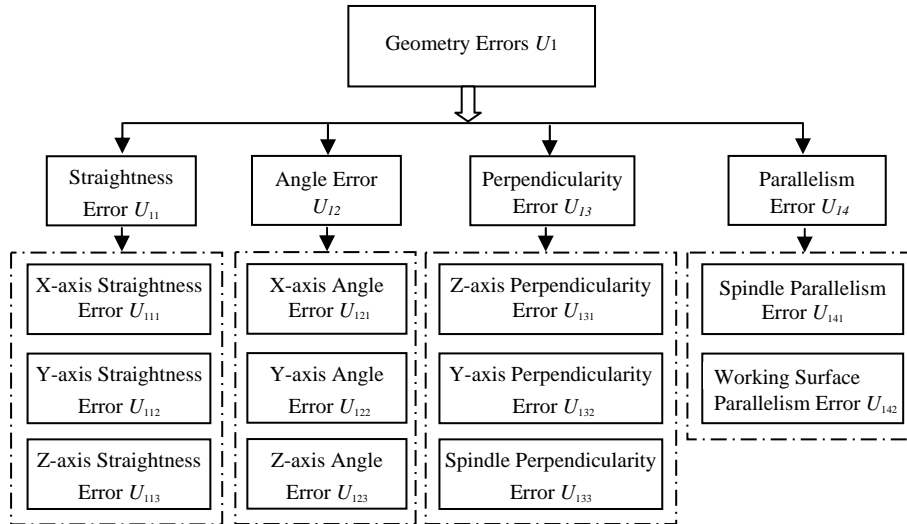


Figure 3. Geometry Error Evaluation Index

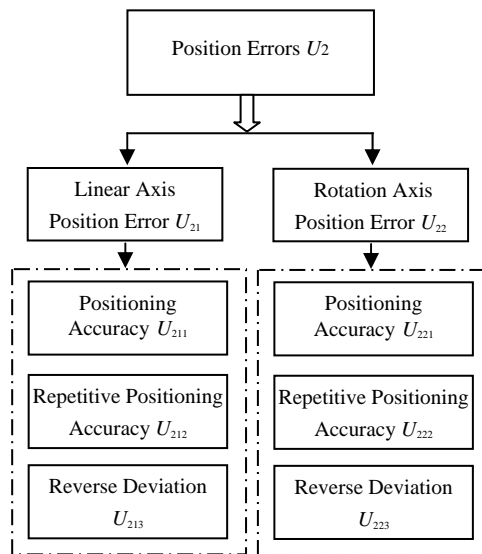


Figure 4. Position Error Evaluation Index

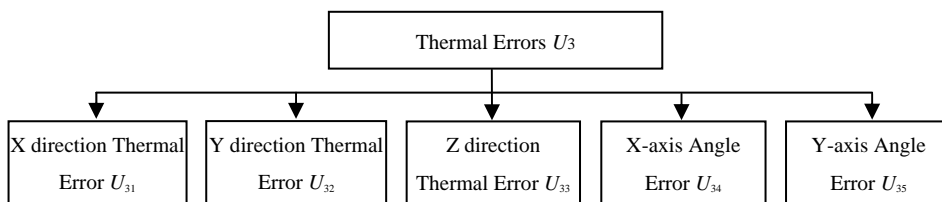


Figure 5. Thermal Error Evaluation Index

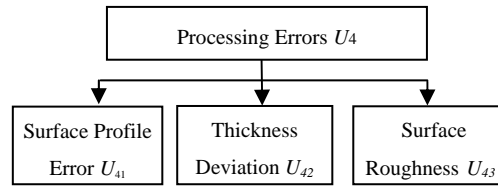


Figure 6. Processing Error Evaluation Index

### 3.2. Constructing Accuracy Evaluation Set of CNC Machine Tools

Evaluation set is evaluators comments of each level evaluation index. According to the characteristics of CNC accuracy evaluation index, evaluation set  $V$  is given. Five ranks are available for the evaluation set, as shown in formula (7).

$$V = \{Excellent, superior, middling, inferior, poor\} \quad (7)$$

### 3.3. Establishing the Priorities of Accuracy Evaluation Indexes

It's subject to determine the indexes priorities. Different methods may eventually lead to the uncertainty of the evaluation results. Thus in practice, regardless of which method to determine the priorities distribution, it should be dependent on a reasonable professional explanation. There're many ways to determine the priorities, including expert scoring method, paired comparison method, correlation coefficient method and entropy weight method, and so on. In this paper, paired comparison method is adopted to determine the priorities distribution.

#### 3.3.1. The Priorities of First Level of CNC Accuracy Evaluation Index System

Some statistical analysis results may provide objective information about priorities distribution. So the priorities of first level of CNC accuracy evaluation index system can be determined according to statistical results of various error sources of CNC [12]. Amended by experts, the priorities  $A$  is as follow:

$$A = [0.25, 0.10, 0.40, 0.25] \quad (8)$$

#### 3.3.2. The Priorities of Second and Third Level of CNC Accuracy Evaluation Index System

Using AHP to determine the priorities of second-level indexes, the most crucial step is to construct a set of pairwise comparison matrices. Each element in an upper level is used to compare the elements in the level immediately below with respect to it [13]. To make comparisons, we need a scale of numbers that indicates how many times more important or dominant one element is over another element with respect to the criterion or property with respect to which they are compared. Table 1 exhibits the scale.

Table 1. Judgement Scale

Intensity of Importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
2	Weak or slight	
3	Moderate importance	Experience and judgement slightly favor one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgement strongly favor one activity over another
6	Strong plus	
7	Demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation

In this paper, the values of matrices are obtained by CNC machine tool experts according to Table 1. Taking thermal errors for example, we can construct judgement matrix  $I_{23}$ , as shown in formula (9).

$$I_{23} = \begin{bmatrix} 1 & 1 & 2 & 5 & 5 \\ 1 & 1 & 2 & 5 & 5 \\ 1/2 & 1/2 & 1 & 7 & 7 \\ 1/5 & 1/5 & 1/7 & 1 & 1 \\ 1/5 & 1/5 & 1/7 & 1 & 1 \end{bmatrix} \tag{9}$$

The priorities, (obtained in exact form by raising the matrix to large powers and summing each row and dividing each by the total sum of all the rows, or approximately by adding each row of the matrix and dividing by their total) are shown in formula (10).

$$A3 = [0.322, 0.322, 0.252, 0.052, 0.052] \tag{10}$$

In the same way, the judgement matrix  $I_{21}$ ,  $I_{22}$  and  $I_{24}$  are constructed as formula 11.

$$I_{21} = \begin{bmatrix} 1 & 4 & 3 & 6 \\ 1/4 & 1 & 3 & 5 \\ 1/3 & 1/3 & 1 & 2 \\ 1/6 & 1/5 & 1/2 & 1 \end{bmatrix} \quad I_{22} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad I_{24} = \begin{bmatrix} 1 & 3 & 6 \\ 1/3 & 1 & 4 \\ 1/6 & 1/4 & 1 \end{bmatrix} \tag{11}$$

The priorities of the second level indexes are shown in Table 2.

**Table 2. Accuracy Index Priorities of CNC Machine Tool**

Global	First Level Indexes	Priorities	Second Level Indexes	Priorities
Accuracy of CNC machine tool	Geometry Errors	0.25	Straightness Error	0.554
			Angle Error	0.258
			Perpendicularity Error	0.124
			Parallelism Error	0.064
	Position Errors	0.10	Linear Axis Position Error	0.5
			Rotation Axis Position Error	0.5
			X direction Thermal Error	0.322
			Y direction Thermal Error	0.322
	Thermal Errors	0.40	Z direction Thermal Error	0.252
			X-axis Angle Error	0.052
			Y-axis Angle Error	0.052
			Surface Profile Error	0.691
	Processing Errors	0.25	Thickness Deviation	0.218
			Surface Roughness	0.091

The priorities of third level indexes can be obtained in the same way.

**3.4. Calculating the Membership of Accuracy Evaluation Indexes**

Membership function is one of the most basic concepts of fuzzy evaluation, so it's very important to determine the membership function. The common methods of determining the membership function include fuzzy statistical method, expert experience method and paired comparison principle [14]. In practice, we usually select membership function in the real number field set based on characteristics of the object, and determine the parameters by experience or empirical data. Common fuzzy distribution includes normal distribution, trapezoidal distribution, S-shaped and parabolic distribution. Parts qualified tolerance is usually represented by the intermediate form trapezoidal distribution [15], which distribution function  $\mu(x)$  is as follows:

$$\mu(x) = \begin{cases} 0 & 0 \leq x \leq a \\ \frac{x-a}{b-a} & a < x \leq b \\ 1 & b < x \leq c \\ \frac{d-x}{d-c} & c < x \leq d \\ 0 & x > d \end{cases} \quad (12)$$

where  $a < b < c < d$

The graph is shown in Figure 7.

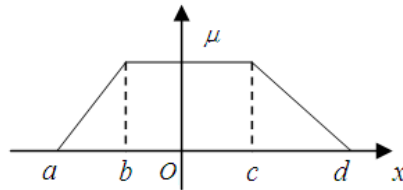


Figure 7. Intermediate Form Trapezoidal Distribution Function

Based on intermediate form trapezoidal distribution function, we can construct the membership function,  $V_1, V_2, V_3, V_4, V_5$ , Where  $x$  denotes the measured value of accuracy indexes.

$$\begin{aligned} V_1 &= [1, 0, 0, 0, 0] & (0 \leq x \leq a) \\ V_2 &= \left[ \frac{x-a}{b-a}, 1 - \frac{x-a}{b-a}, 0, 0, 0 \right] & (a < x \leq b) \\ V_3 &= [0, 0, 1, 0, 0] & (b < x \leq c) \\ V_4 &= \left[ 0, 0, 0, \frac{d-x}{d-c}, 1 - \frac{d-x}{d-c} \right] & (c < x \leq d) \\ V_5 &= [0, 0, 0, 0, 1] & (x > d) \end{aligned} \quad (13)$$

In order to ensure that the evaluation results of CNC machine tool are as objective as possible, it's critical to determine the values of parameter  $a$ ,  $b$ ,  $c$  and  $d$ . Therefore we set up an evaluation group of 50 peoples, which composes of technicians, experts and users. The values of parameters  $a$ ,  $b$ ,  $c$  and  $d$  are determined by them according to experiences.

Taking the index  $U_{111}$  (X-axis Straightness Error) for example, set  $a=0.005$ ,  $b=0.01$ ,  $c=0.015$ ,  $d=0.02$ , when the measured value  $x=0.007$ , according to membership function (13), we can get the valuation vector  $B_i$ , as shown in formula (14).

$$B_i = [0.4, 0.6, 0, 0, 0] \quad (14)$$

#### 4. Applications

The structure of a gantry-type five axis machining center is shown in Figure 8. It's mainly used in mould manufacturing, prototyping, large steel and cast iron mould of automotive industry field or other precision machining field. In this paper, fuzzy comprehensive evaluation method is proposed to evaluate the accuracy of the machining center.

##### 4.1. Constructing the Fuzzy Evaluation Matrixes

According to the membership function of each index in the evaluation system and the actual measured value, we can construct the single factor fuzzy evaluation matrix of each index conveniently.

Taking thermal errors  $U_3$  for example, the indexes of second level are evaluated by fuzzy evaluation method according to the measured values and membership functions. And the fuzzy evaluation results are shown in Table 3.

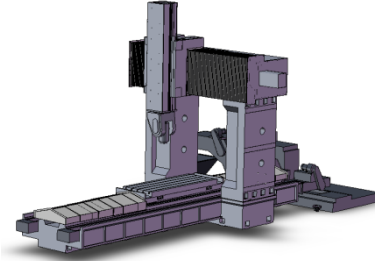


Figure 8. Gantry-type Five Axis Machining Center

Table 3. Evaluation Results of Thermal Errors

First Level	Second Level	Excellent	Superior	Middling	Inferior	Poor
Thermal Errors $U_3$	X direction Thermal Error	0.3	0.7	0	0	0
	Y direction Thermal Error	1	0	0	0	0
	Z direction Thermal Error	0.6	0.4	0	0	0
	X-axis Angle Error	0	0	1	0	0
	Y-axis Angle Error	0.8	0.2	0	0	0

According to the results in Table 3, we can construct the evaluation matrix  $R_3$  of thermal errors, as shown in formula 15.

$$R_3 = \begin{bmatrix} 0.3 & 0.7 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0.6 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 \end{bmatrix} \quad (15)$$

#### 4.2. Fuzzy Comprehensive Evaluation

After single factor fuzzy evaluation matrix is constructed, fuzzy comprehensive vector  $B_i$  of first level can be obtained according to the priorities of second level indexes (as shown in Table 2).

Taking thermal errors  $B_3$  for example.

$$B_3 = A_3 \cdot R_3 = [0.322, 0.322, 0.252, 0.052, 0.052] \cdot \begin{bmatrix} 0.3 & 0.7 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0.6 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 \end{bmatrix}$$

$$= [0.611, 0.337, 0.052, 0, 0]$$

The evaluation result shows that the accuracy of thermal errors is excellent, and the membership is 61.1%.

In the same way, we can get  $B_1$ ,  $B_2$ ,  $B_4$ :

$$B_1 = [0.833, 0.120, 0.047, 0, 0]$$

$$B_2 = [0.625, 0.173, 0.134, 0.068, 0]$$

$$B_4 = [0.765, 0.214, 0.021, 0, 0]$$

(16)



Therefore the comprehensive evaluation vector of the machining center can be obtained, as follows:

$$B=A \cdot R=[0.25,0.10,0.40,0.25] \cdot \begin{bmatrix} 0.833 & 0.120 & 0.047 & 0 & 0 \\ 0.625 & 0.173 & 0.134 & 0.068 & 0 \\ 0.611 & 0.037 & 0.052 & 0 & 0 \\ 0.765 & 0.214 & 0.021 & 0 & 0 \end{bmatrix} \quad (17)$$

$$=[0.706, 0.236, 0.051, 0.007, 0]$$

The evaluation result shows that the comprehensive accuracy of the machining center is excellent, and the membership is 70.6%. Meanwhile, the X-axis angle thermal error is middling, and we should do some error compensation for it.

### 4.3. Analysis of Comprehensive Evaluation Result

This fuzzy comprehensive evaluation results adopt the maximum membership principle. So if there is a maximum of at least two equal, the fuzzy comprehensive evaluation results are invalid, otherwise they are feasible.

From the above evaluation results, we can also analyze the errors source of the CNC machine tool, and puts forward some suggestions to improve the accuracy.

## 5. Conclusion

The paper laid special stress on analyzing the accuracy model of a gantry-type five axis machining center, and established the priorities of accuracy evaluation index system by using APH method. Based on intermediate form trapezoidal distribution function, the membership functions were constructed. Using fuzzy comprehensive evaluation method, the accuracy of the machining center was evaluated, and the results indicated that the comprehensive accuracy of the machining center was excellent.

Compared with the traditional description of the state of the machine accuracy, we can learn about the comprehensive accuracy of CNC machine tools more accurately by fuzzy comprehensive evaluation method. At the same time, the evaluation results enable us to analyze the errors source of the CNC machine tool, and take some suggestions to improve it.

Due to the calculation of fuzzy evaluation method is easy by programming software, and the parameters of membership functions and the priorities of indexes in the evaluation system can be modified continuously according to the empirical data, the subjectivity of fuzzy evaluation method will be reduced in a certain extent. Therefore the comprehensive evaluation method presented in this paper provides a good reference for the accuracy of CNC machine tools.

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