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Quadratic Inverse Function Tsallis Entropy Multi-modulus Blind Equalization Algorithm

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Abstract

In underwater acoustic communication systems, inter-symbol interference (ISI) caused by communication channel distortion is the main factor affecting the quality of communication. Aiming at the shortcomings of computational complexity, slow convergence rate, and poor stability of Multi-Modulus Algorithm (MMA), a quadratic inverse function Tsallis entropy of Cascade Multi-Modulus blind equalization Algorithm (TCMMA) was proposed. In the proposed algorithm, quadratic inverse function Tsallis entropy is fused with Parzen window probability density estimation method, and directly used as the cost function in cascade blind equalizer to reduce the residual mean square error, and the normalized wavelet transform is used to speed up the convergence speed. The simulation results with underwater acoustic channel show the superiority of the proposed algorithm.

Keywords*: multi-modulus blind equalization algorithm, cascade blind equalizer, parzen window, quadratic inverse function, tsallis entropy.*

1. Introduction

Because of the inter-symbol interference(ISI) caused by multipath propagation and channel fading of underwater acoustic communication, the reliability of the underwater data transmission is seriously affected. So, blind equalization techniques are used to eliminate the ISI [1]. In blind equalization algorithms, traditional constant modulus blind equalization algorithm (CMA) has good performance for the signals whose modulus is distributed in a given circle. However, for non-constant modulus signals, the performance of the CMA is poor. Therefore, multi-modulus blind equalization algorithm (MMA) is required [2, 3]. For non-constant modulus signals, the convergence of the MMA can be improved comparison with the CMA and be effective in compensating the ISI introduced by channel and relative deterioration of amplitude. But, the mean square error (Mean Square Error, MSE) of output to the equalizer is larger.

In order to improve the convergence performance of the CMA, according to adaptive theory [4, 5], a cascade blind equalizer cascaded the T/4 fractionally spaced blind equalizer with baud spaced blind equalizer was proposed [6, 7]. More channel information can be obtained by using T/4 fractionally spaced blind equalizer in the first stage and utilizing baud spaced blind equalizer in the second stage which can compensate for channel distortion again and suppress ISI again. The equalizer can effectively accelerate the convergence rate and reduce MSE. The performance of equalization isn’t very satisfactory by using the CMA to update the weight vector of the two equalizers. So, quadratic inverse function Tsallis entropy is used as the cost function in the blind equalizer for better performance [8, 9].

In this paper, quadratic inverse function Tsallis entropy based cascade multi-modulus blind equalization algorithm (TCMMA) is proposed by introducing multi-modulus algorithm and quadratic inverse function Tsallis entropy into the cascade blind equalizer. The proposed algorithm uses the cascade blind equalizer to achieve the second equalization. In the proposed equalizer, the output of T/4 fractionally spaced blind equalizer is used as the input of the baud spaced blind equalizer. The weight vector iteration formula of the two equalizers can be updated in order to get good performance. The input and output of equalizer are divided into real and imaginary parts to overcome channel fading caused by multipath propagation. The convergence

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rate can be improved and the MSE be reduced. Thus, the equalization performance of underwater acoustic channel can be improved.

2. Research Method

2.1. Multi-Modulus Blind Equalization Algorithm

Compared to the CMA, the MMA divides the input and output of equalizer and the error function both into a real part and an imaginary part. They are expressed as follows.

$$\boldsymbol{y}(k)=\boldsymbol{y}_r(k)+j\cdot \boldsymbol{y}_i(k). \quad (1)$$

$$z(k)=z_r(k)+j\cdot z_i(k). \quad (2)$$

$$e(k)=e_r(k)+j\cdot e_i(k). \quad (3)$$

The MMA essentially uses not only the amplitude of input and output signals of the equalizer, but also the phase information. The cost function of the MMA and its iterative relationship of the equalizer tap coefficients are written as

$$J_{\mathrm{MMA}}(f)=E\{(z_r^2(k)-R_r^2(k))^2+(z_i^2(k)-R_i^2(k))^2\}. \quad (4)$$

$$\boldsymbol{f}_r(k+1)=\boldsymbol{f}_r(k)-\mu(e_r(k)+j\cdot e_i(k))\boldsymbol{y}_r^*(k). \quad (5a)$$

$$\boldsymbol{f}(k+1)=\boldsymbol{f}(k)-\mu(e_r(k)+j\cdot e_i(k))\boldsymbol{y}^*(k). \quad (5b)$$

where $R_r^2=\mathrm{E}[|a_r(k)|^4]/\mathrm{E}[|a_i(k)|^2]$ and $R_i^2=\mathrm{E}[|a_r(k)|^4]/\mathrm{E}[|a_i(k)|^2]$. They express the in-phase and quadrature modulus values of the transmitted sequences respectively.

2.2. Cascade Multi-Modulus Blind Equalization Algorithm

For T/4 fractionally spaced blind equalizer, the length of the weight vector is longer than that of channel and its sampling frequency is larger than the Nyquist frequency, the distortion of channel can be compensated and the spectrum aliasing be overcome by subsampling. In this paper, a cascaded blind equalizer is constructed as shown in Figure 1, whose first stage is T/4 fractionally spaced blind equalizer and second stage is baud spaced blind equalizer. When each K switches to “1”, the general cascaded multi-modulus blind equalizer is used. When each K switches to “2”, quadratic inverse function Tsallis entropy based cascade multi-modulus blind equalizer is obtained.

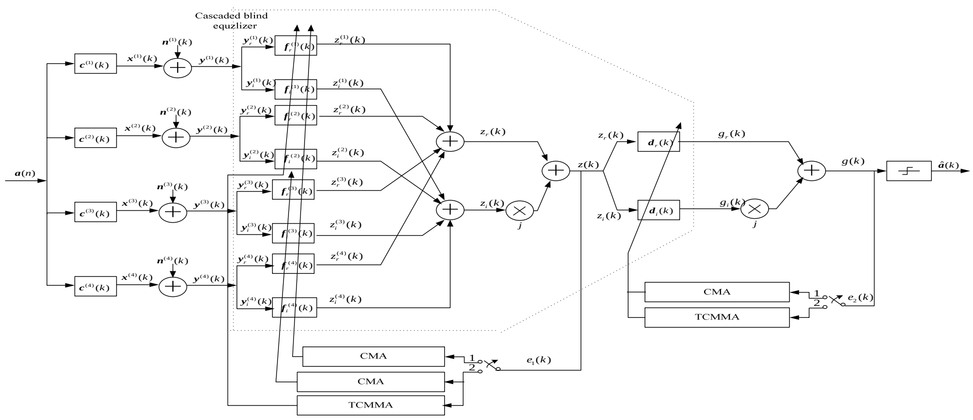


Figure 1. Cascade Multi-modulus Blind Equalizer

In Figure 1, $\boldsymbol{a}(k)$ is the complex transmitted signals, $\boldsymbol{c}^{(p)}(k)$ is the *p*th sub-channel impulse response sequences,and $\boldsymbol{x}^{(p)}(k)$ is the *p*th sub-channel output signal sequences. $\boldsymbol{n}^{(p)}(k)$ is Gaussian white noise sequence (WGN), $\boldsymbol{y}^{(p)}(k)$ is the equalizer input signal vector of the *p*th sub-channel, and $z^{(p)}(k)$ is the equalizer output of the *p*th sub-channel. $z(k)$ is the output vector of the first equalizer, $g(k)$ is the output vector of the second equalizer, and $\boldsymbol{f}^{(p)}(k)$ is the weight vector of the *p*th sub-channel. $\boldsymbol{d}(k)$ is the weight vector of the second filter with length 4*L*, $\hat{\boldsymbol{a}}(k)$ is the estimation of the transmitted signals $\boldsymbol{a}(k)$. $e_1(k)$ and $e_2(k)$ is the iterative error of the first and the second equalizer.

The following equations are given by:

$$\boldsymbol{a}(k)=[a_r(k)+j\cdot a_i(k),a_r(k-1)+j\cdot a_i(k-1),\cdots,$$

$$a_r(k-L-N+1)+j\cdot a_i(k-L-N+1)]^{\mathrm{T}} \tag{6a}$$

$$\boldsymbol{y}^{(p)}(k)=\boldsymbol{y}_r^{(p)}(k)+j\cdot\boldsymbol{y}_i^{(p)}(k). \tag{6b}$$

$$z(k)=z_r(k)+j\cdot z_i(k). \tag{6c}$$

$$g(k)=g_r(k)+j\cdot g_i(k). \tag{6d}$$

$$\boldsymbol{f}^{(p)}(k)=[f_r^{(p)}(1)+j\cdot f_i^{(p)}(1),f_r^{(p)}(2)+j\cdot f_i^{(p)}(2),\cdots,f_r^{(p)}(L)+j\cdot f_i^{(p)}(L)]. \tag{6e}$$

$$e_{1,2}(k)=e_{1,2r}(k)+j\cdot e_{1,2i}(k). \tag{6f}$$

The input vector of the first equalizer is written as:

$$\boldsymbol{y}^{(p)}(k)=\sum_{l=0}^{L-1}c^{(p)}(k)a^{(p)}(k-l)+\boldsymbol{n}^{(p)}(k). \tag{6g}$$

The iteration formula of the cascade multi-modulus equalizer's weight vector is written as:

$$\boldsymbol{f}_r^{(p)}(k+1)=\boldsymbol{f}_r^{(p)}(k)-\mu_1\cdot\boldsymbol{y}_r^{(p)*}(k)e_{1r}(k)z_r^{(p)}(k). \tag{7a}$$

$$\boldsymbol{f}_i^{(p)}(k+1)=\boldsymbol{f}_i^{(p)}(k)-\mu_1\cdot\boldsymbol{y}_i^{(p)*}(k)e_{1i}(k)z_i^{(p)}(k). \tag{7b}$$

$$\boldsymbol{d}_r(k+1)=\boldsymbol{d}_r(k)-\mu_2\cdot\boldsymbol{z}_r^*(k)e_{2r}(k)g_r(k). \tag{8a}$$

$$\boldsymbol{d}_i(k+1)=\boldsymbol{d}_i(k)-\mu_2\cdot\boldsymbol{z}_i^*(k)e_{2i}(k)g_i(k). \tag{8b}$$

$$z_r^{(p)}(k)=\boldsymbol{f}_r^{(p)\mathrm{T}}(k)\boldsymbol{y}_r^{(p)}(k). \tag{9a}$$

$$z_i^{(p)}(k)=\boldsymbol{f}_i^{(p)\mathrm{T}}(k)\boldsymbol{y}_i^{(p)}(k). \tag{9b}$$

$$z_r(k)=\sum_{p=1}^{4}z_r^{(p)}(k). \tag{10a}$$

$$z_i(k) = \sum_{p=1}^{4} z_i^{(p)}(k). \quad (10b)$$

$$g_r(k) = \boldsymbol{d}_r^{\mathrm{T}}(k) \mathrm{z}_r(k). \quad (11a)$$

$$g_i(k) = \boldsymbol{d}_i^{\mathrm{T}}(k) \mathrm{z}_i(k). \quad (11b)$$

$$e_{1r}(k) = z_r(k)(|z_r(k)|^2 - R_r^2). \quad (12a)$$

$$e_{1i}(k) = z_i(k)(|z_i(k)|^2 - R_i^2). \quad (12b)$$

$$e_{2r}(k) = g_r(k)(|g_r(k)|^2 - R_r^2). \quad (13a)$$

$$e_{2i}(k) = g_i(k)(|g_i(k)|^2 - R_i^2). \quad (13b)$$

where L is the length of each equalizer, N is the length of channel, p is the number of branches and $p = 1, 2, 3, 4$. '*'means conjugate, 'T' means transpose, and '^'means estimation. $c^{(p)}(k)$ is the Toeplitz matrix with $L \times (L + N - 1)$ dimensions, $\boldsymbol{z}(k)$ is the signal output vector of the first equalizer. μ_1 and μ_2 are the step size of the first and the second weight vector. Equations (6)-(13) constitute the cascaded multi-modulus blind equalization algorithm (CMMA).

2.3. The Usage of Tsallis Entropy in Cascaded Multi-Modulus Blind Equalization Algorithm

2.3.1. Tsallis Entropy Representation of Equalizer's Weight Vector

Parzen window probability estimation method is the direct use of the sample values to estimate the overall probability density [10], whose basic idea is to use the average density of each point in a certain range to estimate the overall density function. So Parzen window probability estimation method is introduced to solve the problem of probability density estimation in the blind equalization algorithm.

If the probability density $p(x)$ of a point x, the window function should be defined firstly as

$$K(u) = \begin{cases} 1, |u_i| \leq 0.5, i = 1, 2, \cdots, d \\ 0, else \end{cases}. \quad (14)$$

where, $K(\cdot)$ is window function. If h_N is the length of the edge of the super cube, V_N is the volume of d dimension super cube, whose length of the edge is h_N (assume that x is d dimension), and $V_N = h_N^d$.

The basic formula of Parzen window estimation is given as

$$\hat{P}(x) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{V_N} K\left(\frac{x - x_i}{h_N}\right). \quad (15)$$

Where, N is the total number of samples.When quadratic inverse function is used, we have:

$$K(x) = \frac{1}{\sqrt{x^2 + \sigma^2}}. \quad (16)$$

When $d = 1$, the estimation of probability density function $p(x)$ may be written as:

$$\hat{P}(x) = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{\Omega}K\left(\frac{x-x_i}{\Omega}\right) = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{\sqrt{(x-x_i)^2+(\Omega\sigma)^2}} \triangleq \frac{1}{N}\sum_{i=1}^{N}K_{\Omega}(x-x_i). \tag{17}$$

Where

$$K_{\Omega}(x-x_i) = \frac{1}{\sqrt{(x-x_i)^2+(\Omega\sigma)^2}}. \tag{18}$$

where $\Omega = h_N$ is adjustable parameters. So the method of estimate point density of signal x_i can be achieved.

According to the Parzen window estimation method (Parzen window's width is N, namely N sample values), the iterative formula, which uses q order quadratic inverse window function Tsallis entropy as equalizer's weight vector is written as:

$$f(k+1) = f(k) + \mu\frac{\partial S_q(k)}{\partial f}. \tag{19}$$

where, $S_q(k)$ is Tsallis entropy, μ is iterative step-size.

If the quadratic inverse function is used as random variable, X 's q order Tsallis entropy is defined as:

$$S_q(X) = \frac{1}{q-1}(1-\sum p(x)^q),\ q\in(0,1)\cup(1,\infty). \tag{20}$$

Because the entropy of the equalizer output signal is related to its probability density function, the cost function of the quadratic inverse function Tsallis entropy based blind equalization algorithm is defined as:

$$J_q(k) = S_q(|z(k)|^2). \tag{21}$$

$$V_{\alpha}(k) = \int_{-\infty}^{\infty} p^q(z)dz = \sum p(x)^q. \tag{22}$$

where, $p(z)$ is the probability density function of the equalizer output signal $z(k)$. According to Equation (22), the Equation (20) can be rewritten as:

$$S_q(Z) = \frac{1}{q-1}(1-V_q(k)),\ q>0,\ q\neq 1. \tag{23}$$

When $q > 1$, minimizing entropy $S_q(Z)$ is equivalent to maximizing the information potential $V_q(k)$.As long as the information potential of the signal $|z(k)|^2$ is maximum, the cost function of the Equation (21) will be minimum. According to[10, 11], we have:

$$V_q(k) = \int_{-\infty}^{\infty} p^q(z)dz = E\left[p^{q-1}(z)\right]. \tag{24}$$

When the signal is $|z(k)|^2$, its information potential can be represented as:

$$V_q(k) = E[p(|z(k)|^2)^{q-1}] \approx \frac{1}{N}\sum_{j=k+1-N}^{k} p(|z(j)|^2)^{q-1}. \tag{25}$$

In order to maximize the information potential $V_q(k)$, the iterative formula of equalizer weight vector in the Equation (19) can be used.

In the Equation (19), we have:

$$\frac{\partial S_q(k)}{\partial f} = \frac{\partial}{\partial f}\left(\frac{1}{q-1}(1-V_q(k))\right) \tag{26}$$

$$= \frac{-1}{q-1}\cdot\frac{\partial}{\partial f}\left(V_q(k)\right). \ \frac{\partial V_q(k)}{\partial f} = \frac{\partial}{\partial f}\left(\frac{1}{N}\sum_{j=k+1-N}^{k} p(\left|z(j)\right|^2)^{q-1}\right)$$
$$= \frac{1}{N}\sum_{j=k+1-N}^{k}(q-1)\cdot p(|z(j)|^2)^{q-2}\cdot\frac{\partial}{\partial f}(p(|z(j)|^2)). \tag{27}$$

According to Equation (18), we can obtain:

$$K_\Omega(|z(j)|^2 - |z(i)|^2) = \frac{1}{\sqrt{(|z(j)|^2-|z(i)|^2)^2+(\Omega\sigma)^2}}. \tag{28}$$

Then

$$\frac{\partial}{\partial f}\left(p(\left|z(j)\right|^2)\right) \square \frac{1}{N}\sum_{i=1}^{N} K_\Omega(x-x_i) = \frac{1}{N}\frac{\partial}{\partial f}\left(\frac{1}{\sqrt{(|z(j)|^2-|z(i)|^2)^2+(\Omega\sigma)^2}}\right)$$
$$= \frac{-2}{N}\sum_{i=k+1-N}^{k}\left\{\frac{z(j)|^2-|z(i)|^2}{(z(j)|^2-|z(i)|^2)^2+(\Omega\sigma)^2}\cdot K_\Omega(|z(j)|^2\right.$$
$$\left. -|z(i)|^2)[z(j)\cdot \boldsymbol{Y}^*(j) - z(i)\cdot\boldsymbol{Y}^*(i)\right\} \tag{29}$$

According to the Equation (29), the Equation (27) can be rewritten as:

$$\frac{\partial V_q(k)}{\partial f} = \frac{-2(q-1)}{N^2}\sum_{j=k+1-N}^{k}\sum_{i=k+1-N}^{k} p(|z(j)|^2)^{q-2}\cdot\frac{z(j)|^2-|z(i)|^2}{(z(j)|^2-|z(i)|^2)^2+(\Omega\sigma)^2}$$
$$\cdot K_\Omega(|z(j)|^2-|z(i)|^2)[z(j)\cdot\boldsymbol{Y}^*(j)-z(i)\cdot\boldsymbol{Y}^*(i)]\}. \tag{30}$$

According to the Equation (30), the Equation (26) can be rewritten as:

$$\frac{\partial S_q(k)}{\partial f} = \frac{2}{N^2}\sum_{j=k+1-N}^{k}\sum_{i=k+1-N}^{k} p(|z(j)|^2)^{q-2}\cdot\frac{z(j)|^2-|z(i)|^2}{(z(j)|^2-|z(i)|^2)^2+(\Omega\sigma)^2}$$
$$\cdot K_\Omega(|z(j)|^2-|z(i)|^2)[z(j)\cdot\boldsymbol{Y}^*(j)-z(i)\cdot\boldsymbol{Y}^*(i)]\}. \tag{31}$$

where $\alpha = 2, N = 2$. The iteration formula of the equalizer weight vector is given by:

$$f(k+1) = f(k) + \mu\cdot\frac{t\cdot K_\Omega(t)}{2(t^2+(\Omega\sigma)^2)}\cdot[z(k)\cdot\boldsymbol{y}^*(k) - z(k-1)\cdot\boldsymbol{y}^*(k-1)]. \tag{32}$$

Where:

$$t = |z(k)|^2 - |z(k-1)|^2. \tag{33}$$

$$K_{\Omega}(t) = \frac{1}{\sqrt{(t)^2 + (\Omega\sigma)^2}}. \tag{34}$$

where, the step-size μ is the fixed step-size.

2.3.2. The Cascaded Multi-modulus Blind Equalization Algorithm of Quadratic Inverse Function Tsallis Entropy

To overcome the shortcomings of the MMA, such as computational complexity, slow convergence rate and poor stability, the quadratic inverse function Tsallis entropy is used as the cost function in the cascaded multi-modulus blind equalizer, it is shown in Figure 1. In Figure 1, the iteration formulas of each equalizer's first level weight vector are written as

$$\boldsymbol{f}_r^{(1,2)}(k+1)=\boldsymbol{f}_r^{(1,2)}(k)-\mu_1\cdot \boldsymbol{y}_r^{(1,2)*}(k)e_{1r}(k)z_r^{(1,2)}(k). \tag{35a}$$

$$\boldsymbol{f}_i^{(1,2)}(k+1)=\boldsymbol{f}_i^{(1,2)}(k)-\mu_1\cdot \boldsymbol{y}_i^{(1,2)*}(k)e_{1i}(k)z_i^{(1,2)}(k). \tag{35b}$$

$$\boldsymbol{f}_r^{(3,4)}(k+1)=\boldsymbol{f}_r^{(3,4)}(k)+\mu_2\cdot\frac{t_{1r}\cdot K_{\Omega}(t_{1r})}{2(t_{1r}^2+(\Omega\sigma)^2)}\cdot[z_r^{(3,4)}(k)\cdot\boldsymbol{Y}_r^{(3,4)*}(k)-z_r^{(3,4)}(k-1)\cdot\boldsymbol{Y}_r^{(3,4)*}(k-1)]. \tag{36a}$$

$$\boldsymbol{f}_i^{(3,4)}(k+1)=\boldsymbol{f}_i^{(3,4)}(k)+\mu_2\cdot\frac{t_{1i}\cdot K_{\Omega}(t_{1i})}{2(t_{1i}^2+(\Omega\sigma)^2)}\cdot[z(k)\cdot\boldsymbol{y}_r^{(3,4)*}(k)-z_r^{(3,4)}(k-1)\cdot\boldsymbol{y}_r^{(3,4)*}(k-1)]. \tag{36b}$$

$$t_{1r}=|z_r^{(3,4)}(k)|^2-|z_r^{(3,4)}(k-1)|^2. \tag{37a}$$

$$t_{1i}=|z_i^{(3,4)}(k)|^2-|z_i^{(3,4)}(k-1)|^2. \tag{37b}$$

The iteration formulas of the equalizer's second level weight vector is given by:

$$\boldsymbol{d}_r(k+1)=\boldsymbol{d}_r(k)+\mu_3\cdot\frac{t_{2r}\cdot K_{\Omega}(t_{2r})}{2(t_{2r}^2+(\Omega\sigma)^2)}\cdot[g_r(k)\cdot\boldsymbol{z}_r^*(k)-g_r(k-1)\cdot\boldsymbol{z}_r^*(k-1)]. \tag{38a}$$

$$\boldsymbol{d}_i(k+1)=\boldsymbol{d}_i(k)+\mu_3\cdot\frac{t_{2i}\cdot K_{\Omega}(t_{2i})}{2(t_{2i}^2+(\Omega\sigma)^2)}\cdot[g_i(k)\cdot\boldsymbol{z}_i^*(k)-g_i(k-1)\cdot\boldsymbol{z}_i^*(k-1)]. \tag{38b}$$

$$t_{2r}=|g_r(k)|^2-|g_r(k-1)|^2. \tag{39a}$$

$$t_{2i}=|g_i(k)|^2-|g_i(k-1)|^2. \tag{39b}$$

Based on the above analyses, we obtain the quadratic inverse function Tsallis entropy based the cascade multi-modulus blind equalization algorithm (TCMMA).

3. Performance Simulation and Analysis

In order to verify the performance of the proposed algorithm, Multi-Modulus Algorithm (MMA), cascaded Multi-modulus blind equalization algorithm(T/2-CMMA), cascade Multi-modulus blind equalization algorithm (T/4-CMMA) and a quadratic inverse function Tsallis entropy based Cascaded Multi-Modulus blind equalization Algorithm (T/4-TCMMA) are compared. In the experiments, the transmitted signals are 128QAM signal, the signal-to-noise ratio is 30dB.

Table 1. Simulation Parameters

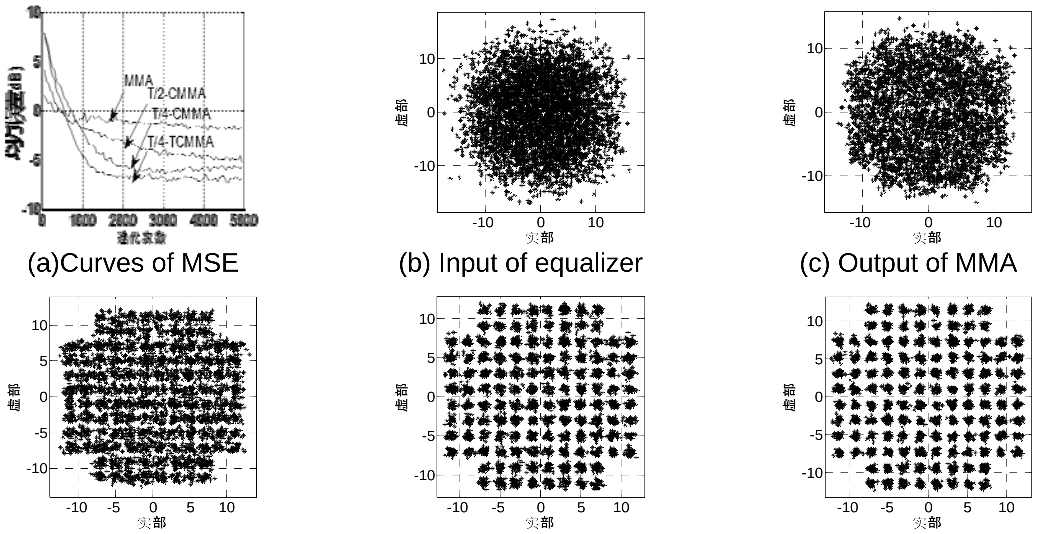
Algorithms	The length of weight vector and its initialization
MMA	The length of weight vector is 32, the sixteenth tap is initialized to 1
T/2-CMMA T/4-CMMA T/4-TCMMA	The length of weight vector of the first equalizer is 8, the fourth tap is initialized to 1, the length of weight vector of the second equalizer is 32, the sixteenth tap is initialized to 1

3.1. Simulation 1

The mixed phase underwater acoustic channel c=[0.3132 -0.1040 0.8908 0.3134] [12], $\Omega\sigma = 1$, the other parameters are shown in Table 2. The results of 500 Monte Carlo simulations are shown in Figure 2.

Table 2. The Other Parameters

Algorithm	step size	step size
MMA	0.5×10^{-6}	
T/2-CMMA	0.5×10^{-6}	0.1×10^{-7}
T/4-CMMA	0.53×10^{-6}	0.13×10^{-7}
T/4-TCMMA	0.4×10^{-5}	0.1×10^{-7}



(a)Curves of MSE (b) Input of equalizer (c) Output of MMA

(d) Output of T/2-CMMA (e) Output of T/4-CMMA (f) Output of T/4-CMMA

Figure 2. The Results of Simulation

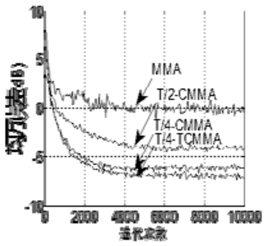
As shown in Figure 2(a), the convergence speed of the T/4-TCMMA has an improvement of about 1000 steps comparison with T/4-CMMA. When the T/4-CMMA and T/4-TCMMA converge, the MMA and the T/2-CMMA still diverge. The steady-state mean square error of the T/4-TCMMA is the smallest and about -6dB. So the proposed algorithm has better performance.

3.2. Simulation 2

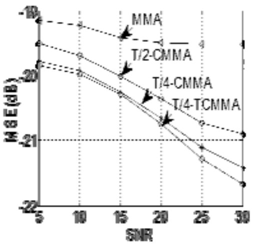
The deep spectral zero points underwater acoustic channel c=[0.2443 0.1183 -0.0455 -0.0905 0.6766 0.6622 -0.1163 0.0786] [13], $\Omega\sigma = 1$. The other parameters are shown in Table 3. The results of 500 Monte Carlo simulations are shown in Figure 3.

Table 3. The Other Parameters

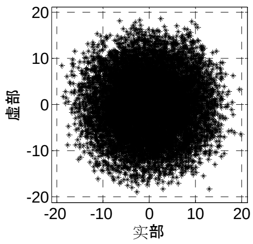
Algorithm	step size	step size
MMA	0.5×10^{-6}	
T/2-CMMA	0.5×10^{-6}	0.1×10^{-7}
T/4-CMMA	0.1×10^{-5}	0.1×10^{-7}
T/4-TCMMA	0.8×10^{-6}	0.8×10^{-6}



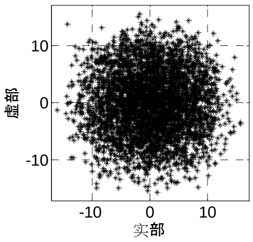
(a) Curves of MSE



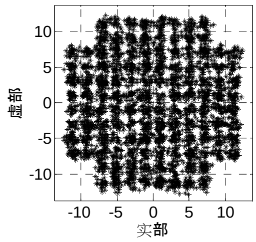
(b) Curves of MSE with SNR



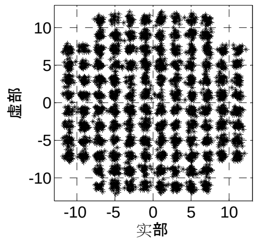
(c) Input of equalizer



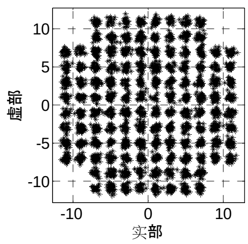
(d) Output of MMA



(e) Output of T/2-CMMA



(f) Output of T/4-CMMA



(g) Output of T/4-TCMMA

Figure 3. The Results of Simulation

As shown in Figure 3(a), the convergence speed of T/4-TCMMA has an improvement of about 1000 steps and 1000 steps comparison with the MMA and the T/2-CMMA, respectively. When the T/4-CMMA and T/4-TCMMA converge, the T/2-CMMA still diverges. The steady-state mean square error T/4-TCMMA is the smallest and about -7dB. With the increasing SNR, the MSE of T/4-TCMMA has the fastest reducing rate. So the proposed algorithm has better performance.

4. Conclusion

Aiming at the shortcomings of computational complexity, slow convergence rate, and poor stability of multi-modulus algorithm, a quadratic inverse function Tsallis entropy based cascaded multi-modulus blind equalization algorithm (TCMMA) is proposed by introducing multi-modulus blind equalization algorithm and cascade multi-modulus blind equalizer. This proposed algorithm designs a cascade blind equalizer, whose first stage is T/4 fractionally spaced blind equalizer and second stage is baud spaced blind equalizer, to implement secondary equalization. The weight vector of two equalizer use quadratic inverse function Tsallis entropy as the cost function to get a better equalizer structure and the input and output of equalizer are divided into real and imaginary part through multi-modulus ideas. In this way, the convergence speed can be accelerated and the MSE can be reduced. Compared to the MMA, T/2-CMMA, and T/4-CMMA, this proposed algorithm not only can effectively overcome ISI and improvment convergence speed, but also has smaller steady-state mean square error and more clearer and compact constellations. Simulation results with Underwater acoustic channel demonstrate the effectiveness of the proposed algorithm.

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