Application of Support Vector Machine to Reliability Analysis of Engine Systems

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Abstract
Reliability analysis plays a very important role in assessing product performances and making maintenance plans for maintainable production. To find effective ways of forecasting the reliability in engine systems, the paper presents a comparative study on the prediction performances using support vector machine (SVM), least square support vector machine (LSSVM) and neural network. The reliability indexes of engine systems are computed using the Weibull probability paper programmed with Matlab. Illustrative examples show that probability distributions of forecasting outcomes using different methods are consistent to the actual probability distribution. And the two methods of SVM and LSSVM can provide the accurate predictions of the characteristic life, so SVM and LSSVM are both effective prediction methods for reliability analysis in engine systems. Moreover, the predictive precision based on LSSVM is higher than that based on SVM, especially in small samples. Because of its lower computation costs and higher precision, the reliability prediction using LSSVM is more popular.

Keywords: reliability analysis, support vector machines, least square support vector machine, neural network, learning methods

1. Introduction
        The ability of accurately predicting reliability for engine systemsis an invaluable asset for many manufacturing companies. Especially for automobile production, the main concern is to satisfy the increasing demands from customers and conform to stricter acts and regulations by governments [1]. As system reliability indexes vary with time, it is not easy to predict it accurately.
        There are many forecasting techniques about time series. Traditionally, the methods based on stochastic process theory have been developed and used widely for assessing the system reliability during the whole lifetime [2, 3]. But they impose some restrictions on failure classes, so it's difficult to satisfy and validate all the assumptions. In practice, the way of simplifying them is often used.
        Neural networks are universal function approximators that can map any non-linear function without a priori assumptions about the properties of the data [4]. The theory is more powerful in describing the dynamics of reliability in comparison to traditional statistical models. With the emergence and development of statistical learning theory, the research of application of neural networks in reliability engineering has been made gradually. Zheng [5] illustrated a non-parametric software reliability prediction system based on neural network ensembles. And it improved the system predictability by combing multiple neural networks. Lolas and Olatunbosun [6] demonstrated how a tool like neural networks can be designed and optimized for use in reliability performance predictions. Xu et al. [1] applied Radial Basis Function (RBF) neural networks to forecast engine system reliability. The comparative study among feed-forward multilayer perceptron (MLP) and RBF was presented. The results showed that the model is more accurate than those.
        Another learning method is support vector machine (SVM) [7-10] proposed by Vapnik, which is based on the structured risk minimization (SRM) principle and statistical learning theory. SVM has better generalization performance than other neural network models. The solution of SVM is unique, optimal and absent from local minima, unlike other networks' training

which requires non-linear optimization thus running the danger of getting stuck in a local minima. However, the computation cost of SVM is very high. Suykens et al. [11] introduced the modified least squares loss function into SVM, which is known as the least square support machine (LSSVM). Unlike SVM, LSSVM turns inequality constraints into equality constraints, which makes computation efficiency higher. At the same time, LSSVM considers the training errors of all the training samples. So far SVMs were successfully applied in many fields, such as pattern recognition problems, function estimation, time series forecasting, disease detecting in medicine [12, 13]. But the research on the application of SVM and LSSVM in reliability prediction is very limited [14, 15].

This paper aims at validating the effectiveness of SVM and LSSVM for the reliability prediction of engine systems. The comparative study of the predicted results of the engine systems by SVM, LSSVM, MLP, and RBF is made. Based on the prediction, the affection on the life characteristics of the engine system is analyzed.

2. SVM and LSSVM
2.1 SVM

Let data set $T = \{(x_i, y_i)\}$, where $x_i \in R^n$, $y_i \in R$, $i = 1, \cdots, l$. For regression analysis in SVM, there are two basic objectives. The first is to find real approximating function $f(x)$, which makes the structure risk of the function estimation lest in the insensitive loss function ε. The second is to make function $f(x)$ flat.

2.1.1 Liner Support Vector Machine

When the relaiton of the iput and the output data is linear, the regression function can be shown in Eq. (1).

$$f(x, w) = w \cdot x + b \tag{1}$$

To make the function flat, the parameter w in Eq.(1) should be at most less. So the solution is transformed into the following optimazation problem.

$$\min \frac{1}{2} \| w \|^2$$
$$s.t. \qquad | y_i - (w \cdot x) - b | \le \varepsilon, \quad i = 1, 2, \cdots l \tag{2}$$

Considering the possible errors and introducing two slack variables ξ_i, ξ_i^*, and penalty parameter C, the above optimazation objective function can be written as shown in Eq.(3)[8]:

$$\min \frac{1}{2} \| w \|^2 + C \sum_{i=1}^{l} (\xi_i + \xi_i^*)$$
$$s.t. \quad \begin{cases} y_i - <w \cdot x_i> -b \le \varepsilon + \xi_i^* \\ <w \cdot x_i> +b - y_i \le \varepsilon + \xi_i \\ \xi_i, \xi_i^* \ge 0, \quad i = 1, 2, \cdots l \end{cases} \tag{3}$$

The solution method is commonly Lagrange Multiplier technique. So the dual form of the initial optimazation question is expressed as follow.

$$\min \frac{1}{2} \sum_{i,j=1}^{l} (\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*) <x_i \cdot x_j> + \varepsilon \sum_{i=1}^{l} (\alpha_i + \alpha_i^*) - \sum_{i=1}^{l} y_i (\alpha_i - \alpha_i^*)$$
$$s.t. \quad \begin{cases} \sum_{i=1}^{l} (\alpha_i - \alpha_i^*) = 0 \\ (\alpha_i - \alpha_i^*) \in [0, C], \quad i = 1, 2, \cdots l \end{cases} \tag{4}$$

Further, from Eq. (4), we can obtain :

$$w = \sum_{i=1}^{l} (\alpha_i^* - \alpha_i) x_i \tag{5}$$

Submitting Eq. (5) into Eq.(1), the linear regression equation is followed.

$$f(x) = \sum_{i=1}^{l} (\alpha_i^* - \alpha_i) <x_i \cdot x> + b \tag{6}$$

2.1.2 Nonlinear Support Vector Machine

If any agrithm can be expressed by the dot product, its generalization form can be achived by kernel functions. When the relation of the input and output data is nonlinear, the nonlinear function approximations can be got by replacing the dot product of input vectors with a kernel function. It can be represented by $k(x, x')$, where x and x' are each input vectors.

Replacing the dot product of input vectors in Eq.(4) with the kernel fucntion, we obtain

$$\min \frac{1}{2} \sum_{i,j=1}^{l} (\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*) k(x_i \cdot x_j) + \varepsilon \sum_{i=1}^{l} (\alpha_i + \alpha_i^*) - \sum_{i=1}^{l} y_i (\alpha_i - \alpha_i^*)$$
$$s.t. \quad \sum_{i=1}^{l} (\alpha_i - \alpha_i^*) = 0, \qquad \alpha_i, \alpha_i^* \in [0, C], \quad i = 1, 2, \cdots l \tag{7}$$

At the same method, replacing the dot product of input vectors in Eq.(6) with the kernel fucntion, the nonlinear regression equation is following.

$$f(x) = \sum_{i=1}^{l} (\alpha_i^* - \alpha_i) K(x_i \cdot x) + b \tag{8}$$

2.2 LSSVM

According to the LSSVM theory, the data set can be written as shown in Eq.(9).

$$f(x) = \varphi(x)\omega + b \tag{9}$$

Where ω denotes the weight vector; φ represents the nonlinear function that maps the input space to a high-dimension feature space and performs linear regression; and b is the bias term.

Unlike SVM, LSSVM turns inequality constraints into the equality. For function estimation, the original optimization problem, consequently, changes according to the SRM principle. The algorithm is the solution to convex quadratic programming, as following formula.

$$\min \frac{1}{2} \| \omega \|^2 + \frac{\gamma}{2} \sum_{i=1}^{l} \eta_i^2$$
$$s.t. \quad y_i - (<w \cdot \varphi(x_i)> + b) = \eta_i, \quad i = 1, 2, \cdots l \tag{10}$$

Where γ denotes the regularization constant, η_i represents the data errors.

The solution method is commonly Lagrange Multiplier technique. So the Lagrange polynomial is shown as Eq. (11).

$$L(\omega, \alpha, b, \eta) = \frac{1}{2} \| \omega \|^2 + \frac{\gamma}{2} \sum_{i=1}^{l} \eta_i^2 - \sum_{i=1}^{n} \alpha_i \left((\varphi(x_i)\omega) + b + \eta_i - y_i \right) \tag{11}$$

Further, we can obtain the set of linear equations (12).

$$\begin{bmatrix} I & 0 & 0 & -Z^{\mathrm{T}} \\ 0 & 0 & 0 & -1^{\mathrm{T}} \\ 0 & 0 & \gamma I & -I \\ Z & 1 & I & 0 \end{bmatrix} \begin{bmatrix} \omega \\ b \\ \eta \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ y \end{bmatrix} \tag{12}$$

where $Z = [\varphi(x_1) \dots \varphi(x_l)]^T$, $1 = [1 \dots 1]^T$, $y = [y_1 \dots y_l]^T$, $\eta = [\eta_1 \dots \eta_l]^T$, $\alpha = [\alpha_1 \dots \alpha_l]^T$
The solution is also expressed as Eq.(13)

$$\begin{bmatrix} 0 & 1^{\mathrm{T}} \\ 1 & K + \dfrac{\gamma}{I} \end{bmatrix} \begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix} \tag{13}$$

The solution to Eq. (13) is given by Eq.(14).

$$\begin{cases} \alpha = A^{-1}(y - b1) \\ b = \dfrac{1^{\mathrm{T}} A^{-1} y}{1^{\mathrm{T}} A^{-1} 1} \end{cases} \tag{14}$$

In Eq. (14), $A = K + \gamma / I$.

Submitting Eq. (14) into $\omega = \sum\limits_{i=1}^{l} \alpha_i x_i$, in accordance with Mercer condition, the regression equation is followed.

$$f(x) = \sum_{i=1}^{l} \alpha_i K(x_i \cdot x) + b \tag{15}$$

Where $K(x_i \cdot x)$ is called the kernel functions. Eq. (15) is the desired LSSVM model.

3 Reliability Analysis of Engine Systems Using SVM and LSSVM

System reliability is the function varying with time. For engine systems, the time can be time between failures, time to failure, or the total failure numbers, which can be considered as a collection of random variables. So reliability predictive can be finished by the traditional time series analysis method.

This paper proposes the application of SVM and LSSVM described in section 2 to predict system reliability. And the comparative study of algorithm performance among SVM, LSSVM and neural networks is made. In order to assess the predictive errors among the above methods, the normalized root mean square error measure (NRMSE) is introduced.

$$NRMS = \sqrt{\frac{\sum (x(t) - \hat{x}(t))^2}{\sum x^2(t)}} \tag{16}$$

In Eq.(16), $\hat{x}(t)$ denotes the prediction of $x(t)$.

The prediction is classified into the long-term and short-term. Due to the accumulation of errors, the performance of the former is poorer than that one of the latter. For engine system, the short term predictive results are more effective, which provide timely information for preventive maintenance and corrective maintenance plans. So here only the single-step-ahead predictions will be considered.

For the reliability prediction of engine systems by SVM and LSSVM,it's important toselect the suit kernel function.As kernel functions satisfy the Mercer condition,they enable thedot product to be computed in high-dimension space using low-dimension space data input without the transfer function ϕ [16]. We will make the selection of the radial basis function (RBF) as the kernel function, which is commonly useful in function estimation. The RBF kernel function is represented as Eq.(17).

$$K(x_i, x_j) = \exp(-\frac{\|x_i - x_j\|^2}{2\sigma^2})$$ (17)

In Eq.(17), σ is the kernel function parameter.

All data are divided into training data and predictive data. To obtain the optimal parameter combination (γ, σ) in establishing the SVM and LSSVM models, this research used the grid search algorithm with a *k*-fold cross-validation method [17]. Further, the SVM and LSSVM predictive models will be built with the optimal combination. The predictive data are substituted into the model; the reliability prediction will be computed.

3.1 Reliability Prediction of Engine Systems by SVM and LSSVM

Table 1 [1] gives the original test data of time to failure for 40 suits of turbochargers. The first column in Table 1 denotes the failure order. The second column in Table 1 denotes time to failure of the turbochargers.

Commonly, the estimations of reliability like this are achieved by median ranking. The formula [18] is as follows.

$$R(T_i) = 1 - \frac{i - 0.3}{n + 0.4}$$ (18)

Table 1. Failure data and reliability of turbocharges

Failure Order (i)	Timetofailure ($T/1000h$)	$R(T_i)$ (%)	Failure Order (i)	Timetofailure ($T/1000h$)	$R(T_i)$ (%)
1	1.6	99.303	21	6.5	79.382
2	2	98.307	22	6.7	78.386
3	2.6	97.311	23	7	77.390
4	3	96.315	24	7.1	76.394
5	3.5	95.319	25	7.3	75.398
6	3.9	94.323	26	7.3	74.402
7	4.5	93.327	27	7.3	73.406
8	4.6	92.331	28	7.7	72.410
9	4.8	91.335	29	7.7	71.414
10	5	90.339	30	7.8	70.418
11	5.1	89.343	31	7.9	69.422
12	5.3	88.347	32	8	68.426
13	5.4	87.351	33	8.1	67.430
14	5.6	86.355	34	8.3	66.434
15	5.8	85.359	35	8.4	65.438
16	6	84.363	36	8.4	64.442
17	6	83.367	37	8.5	63.446
18	6.1	82.371	38	8.7	62.450
19	6.3	81.375	39	8.8	61.454
20	6.5	80.378	40	9	60.458

Where i is failure order.

Substituting the failure orders in table 1 into Eq.(18), the corresponding reliability will be obtained. The results are listed in the third column in Table 1.

All the data are divided into training data and predictive data. The single-step-ahead predictions are adopted. The number of the lagged variables is 35. Initially, the former 35 data set of time to failure and $R(T_i)$ are considered as the training data; the latter one are viewed as predictive data. The time to failure is substituted into the SVM and LSSVM model, and the reliability predictionsare computed. Other predictive results are attainted with the same method.

The results are tabulated in the sixth and seventh column in table 2. The figure of the reliability of the turbocharger training and predictive results is shown as Figure 1(a) and Figure1 (b).

To evaluate the performance of SVM and LSSVM, the predictive results with othermethods, such as MLP (logistic activation), MLP (Gaussian activation), and RBF, are tabulated into table 2. For the details about their computation process see Xu K et al.[1].

Substituting the predictive results into Eq.(16), their perspective NRMS are attained, which are tabulated in table 2. This shows that predictive results by SVM and LSSVM can be approved with compare of the other results. So reliability prediction of engine system by SVM and LSSVM is effective.
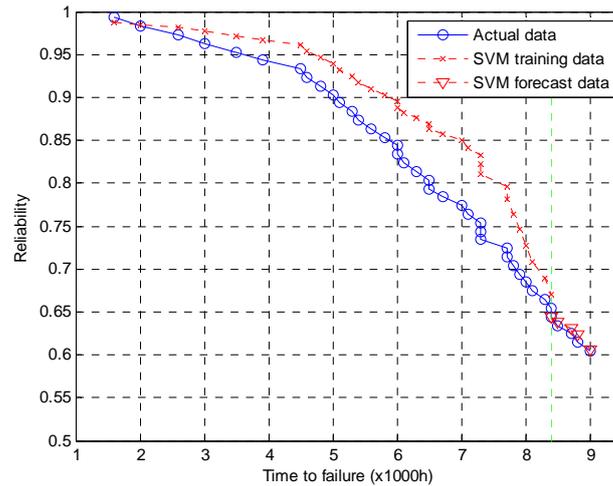
Figure 1(a). Reliability of the turbocharger training and predictive results with SVM
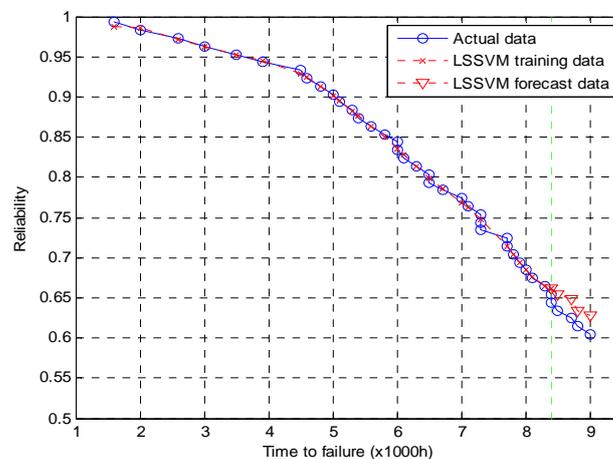

Figure 1(b). Reliability of the turbocharger training and predictive results with LSSVM

Table 2. Forecasting result of turbochargers reliability using different method

Number	Reliability (actual)/%	MLP (logistic activation)/%	MLP (Gaussian activation)/%	RBF (Gaussian activation)/%	SVM /%	LSSVM /%
36	64.442	66.01	65.39	64.40	64.37	66.190
37	63.446	65.42	64.76	63.31	63.86	65.480
38	62.450	64.71	64.41	62.14	63.14	64.830
39	61.454	64.19	63.89	61.10	62.36	63.420
40	60.458	63.57	63.60	60.04	61.65	62.770
NRMS		0.0383	0.0338	0.0046	0.0088	0.0336

3.2   Reliability Analysis of Engine Systems .

In order to verify the effect of the predictive results on reliability indexes, probability distribution needs to be determined. Weibull reliability paper is plotted in Matlab 7.0. The actual reliability and the predictive results by SVM and LSSVM are analyzed with the help of it. The data process results show that their distributions are consistent, which both follow weibull failure distribution. For details see the Figure 3, Figure 4, and Figure 5.
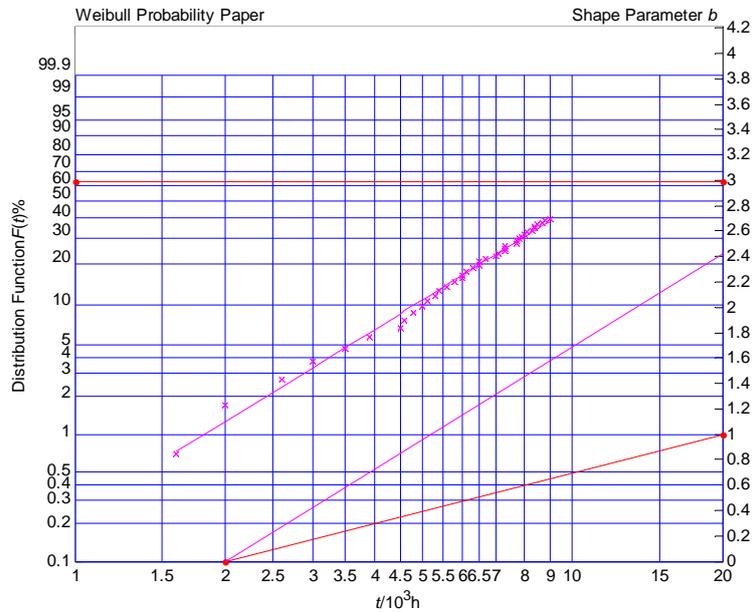

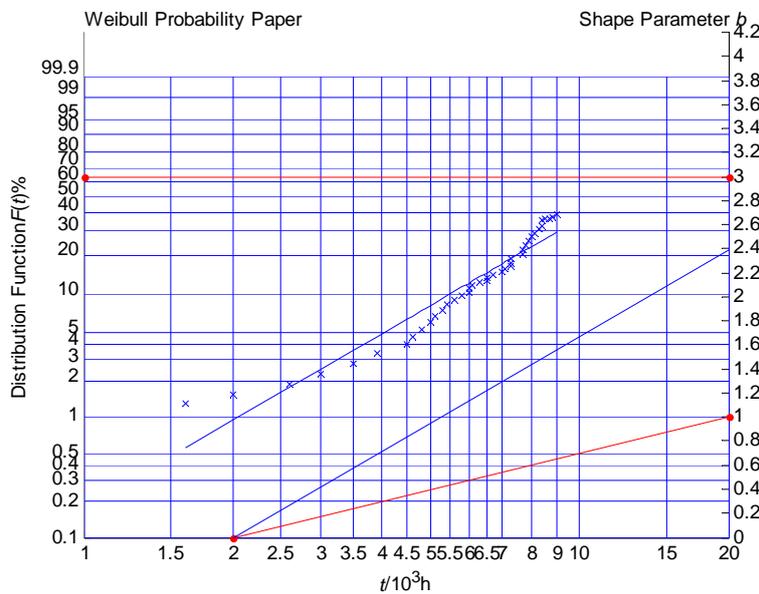Figure 3. Reliability analysis of turbochargers (actual data)

Figure 3. Reliability analysis of turbochargers (forecast data by SVM)

The shape parameters and characteristic life of both actual data and the predictive results by SVM and LSSVM are shown as Table 3. Their goodnesses of fit are all 1.

The relative error between the actual reliability and the predictive reliability by SVM is only 14.9%. The relative error between the actual reliability and the predictive reliability by LSSVM is only 4.1%, which shows that the predictive results are more perfect.
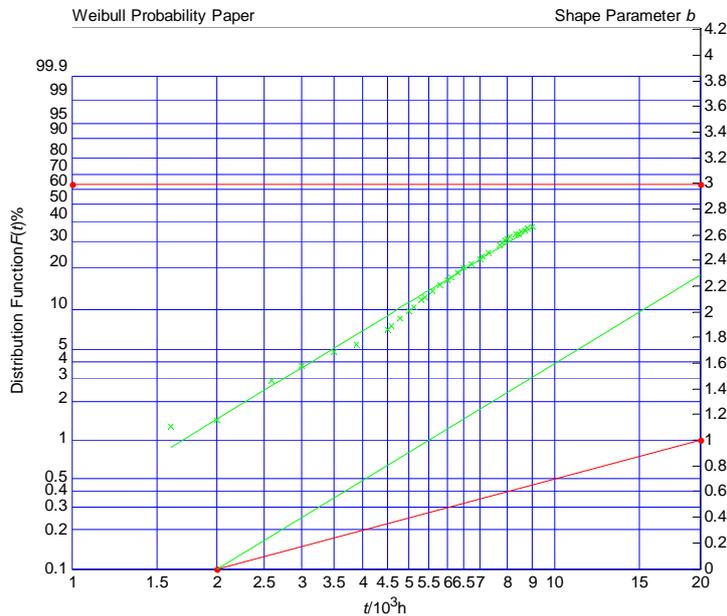

Figure 5. Reliability analysis of turbochargers (forecast data by LSSVM)

Table 3. Reliability analysis result by weibull probability paper

	b	$T(\times 10^3 h)$	Goodnessof fit
Actual reliability	2.423	12.155	1
Predictivereliability by SVM	2.397	13.966	1
Predictivereliability by LSSVM	2.290	12.653	1

4   Conclusions

This research applied the SVM and LSSVM in the reliability prediction of engine systems. The predictive performance of the SVM and LSSVM were compared with that of the neural networks of MPL (logistic activation), MLP (Gaussian activation), and RBF. The simulation experiment outcomes show that the predictive results by SVM and LSSVM are perfect. The failure distribution analysis of the predictive reliabilities of engine systems with SVM and LSSVM was close to that of the actual. So SVM and LSSVM are the alternative choices of the reliability prediction of engine systems.

The numerical results also show that the predict precise of the method based on LSSVM is higher than that of SVM. Especially in small samples, the prediction by LSSVM will be more popular, because its compution cost is lower and the precise can be more satisfied.

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