

The Application of Maximum Principle in Supply Chain Cost Optimization

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Abstract

In this paper, using the maximum principle for analyzing dynamic cost, we propose a new two-stage supply chain model of the manufacturing-assembly mode for high-tech perishable products supply chain and obtain the optimal conditions and results. On this basis, we further research the effect of localization of CODP on the total cost and the relation of CODP, inventory policy and demand type through the data simulation. The results of simulation show that CODP locates in the downstream of the product life cycle, is a linear function of the product life cycle. The result indicates that the demand forecast is the main factors influencing the total cost; meanwhile the mode of production according to the demand forecast is the deciding factor of the total cost. Also the model can reflect the relation between the total cost of two-stage supply chain and inventory, demand.

Keywords: maximum principle, cost, optimization

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1. Introduction

Compared with the traditional products, perishable products have a shorter product life cycle and quicker update speed, such as clothing, PC industry, etc. Especially high-tech perishable products are more prominent [1]. Because of the shorter product life cycle, the value of high-tech perishable products decreases linearly and the supply chain model has its particularity [2]. Characteristics of high-tech perishable products determine its supply chain is suitable for mass customization (MC) [3]. Customer order decoupling point (CODP) is a conversion point in the supply chain from the mass production to the personalized customization, its position in the supply chain effects production rate and inventory level and is determined by inventory forecast and demand jointly [4].

Holt et al. proposed quadratic objective function to describe the total cost of two-stage production environment which is called HMMS model [5]. On this basis, Olhager considered a P/D (production lead time/delivery lead time) ratio and the related demand volatility for positioning of the customer order decoupling point [6]. Kundu et al. put forward a knowledge-based approach in determining of the CODP and considered the tradeoff between the physical efficiency of the supply chain and the market responsiveness [7]. Imre Dobos found optimal inventory policies in a reverse logistics system with special structure. The total costs of this system consist of the quadratic holding costs for these two stores and the quadratic manufacturing, remanufacturing and disposal costs [8]. Rao Kai et al. proposed a basic model of production cost optimization and its extension in manufacturer implementation postponement strategy in mass customization [9]. Imre Dobos discussed the bullwhip effect of supply chain based HMMS model and gave an extended HMMS model for decreasing the bullwhip effect [10]. Wang feng et al. presented a new production model with the idea of different CODP for different products based on the analysis of the deficiency of single CODP in mass customization [11]. Dan bin et al. assumed a supply chain which included one manufacturer and one supplier and constructed a cost optimization model of supply chain for implementing postponement strategy in mass customization [12]. Li dan-dan et al. improved a CODP decision-making model based on the queuing theory model considered the cost optimization [13]. In-Jae Jeong proposed a dynamic mode I to simultaneously determine the CODP and production-inventory plan in a supply chain considering that production rate was a constant based the HMMS model

[14]. About the inventory policy, MonamiDas Roy et al. researched the relationship of producer-buyer in zero-inventory policy [15].

Raw materials and accessories of high-tech perishable products have high level of generalization which coupling degree of the manufacturing and assembly process is low. In the supply chain, customer demand has frequent changes and quick transformation which determines its CODP is between the manufacturing and the assembly process that is a more reasonable choice. In this paper, we simplify the structure of the supply chain for high-tech perishable products according to its characteristics, which be divided into the two-stage model of manufacturing and assembly. Using dynamic analysis method, we analyze position of CODP based on optimal cost. Figure 1 shows structure of the two-stage supply chain of high-tech perishable products.

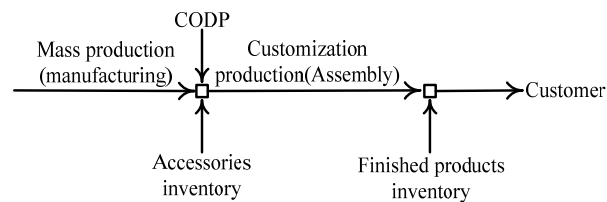


Figure 1. The Two-stage Supply Chain of High-tech Perishable Products

2. Research Method

We suppose there are two stages in the supply chain which are manufacturing and assembly process and only consider the one-way flow of single product. The inventory in the supply chain includes two points of accessories and finished products. The two-stage supply chain model of high-tech perishable production is depicted in Figure 2.

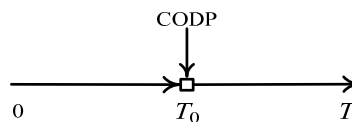


Figure 2. Mathematical Model of the Two-stage Supply Chain

In the model, The following parameters are used in the model:

T_0 : position of CODP,
 T : product life cycle,

\bar{P}_1 : target production rate during manufacturing,
 $P_1(t)$: production rate at time t during manufacturing,
 c_1 : constant cost per unit deviation from target production rate during manufacturing,

\bar{I}_1 : target inventory level during manufacturing,
 $I_1(t)$: inventory level at time t during manufacturing,
 c_2 : constant cost per unit deviation from target inventory level during manufacturing,

\bar{P}_2 : target production rate during assembly,
 $P_2(t)$: production rate at time t during assembly,
 c_3 : constant cost per unit deviation from target production rate during assembly,

\bar{I}_2 : target inventory level during assembly,
 $I_2(t)$: inventory level at time t during assembly,
 c_4 : constant cost per unit deviation from target inventory level during assembly,

$Q_1(t)$: demand from assembly process,
 $Q_2(t)$: demand from terminal customer process.

The objective function of minimizing total cost of the two-stage supply chain we can give according to Figure 2 as follows:

$$f = \min \int_0^T \left\{ c_1 (P_1(t) - \bar{P}_1)^2 + c_2 (I_1(t) - \bar{I}_1)^2 \right\} dt + \min \int_0^T \left\{ c_3 (P_2(t) - \bar{P}_2)^2 + c_4 (I_2(t) - \bar{I}_2)^2 \right\} dt \quad (1)$$

$$\text{s.t. } \dot{I}_1(t) = P_1(t) - \dot{Q}_1(t) \quad (2)$$

$$\dot{I}_2(t) = P_2(t) - \dot{Q}_2(t) \quad (3)$$

$$I_1(t) \geq 0 \quad I_2(t) \geq 0$$

3. Results and Analysis

Considering the relativity between perishable production and time, we assume that the demand from assembly process is a time-dependent constant. We can give an equation:

$$Q_1(t) = q_1 t \quad (4)$$

The same equation is as follows:

$$Q_1(t) = q_1 t \quad (5)$$

Applying the maximum principle of Pontryagin, we obtain the following objective function:

$$L = -c_1 (P_1(t) - \bar{P}_1)^2 - c_2 (I_1(t) - \bar{I}_1)^2 - c_3 (P_2(t) - \bar{P}_2)^2 - c_4 (I_2(t) - \bar{I}_2)^2 \quad (6)$$

The Hamilton function of the objective function can be formulized as follows:

$$H(P_1(t), I_1(t), P_2(t), I_2(t)) = -c_1 (P_1(t) - \bar{P}_1)^2 - c_2 (I_1(t) - \bar{I}_1)^2 - c_3 (P_2(t) - \bar{P}_2)^2 - c_4 (I_2(t) - \bar{I}_2)^2 + \varphi_1(t)(P_1(t) - \dot{I}_1(t)) + \varphi_2(t)(P_2(t) - \dot{I}_2(t)) \quad (7)$$

Assume optimal parameters as $(P_1^0(t), I_1^0(t), P_2^0(t), I_2^0(t))$, where for $0 \leq t \leq T$, we can have $\varphi_1(t) \neq 0$, $\varphi_2(t) \neq 0$. The first derivative of (7) on $I_1(t)$, $I_2(t)$, $P_1(t)$, $P_2(t)$ respectively is as follows:

$$\frac{\partial H(P_1^0(t), I_1^0(t), P_2^0(t), I_2^0(t), t)}{\partial I_1(t)} = -2c_2 (I_1^0(t) - \bar{I}_1) = -\dot{\varphi}_1(t) \quad (8)$$

$$\frac{\partial H(P_1^0(t), I_1^0(t), P_2^0(t), I_2^0(t), t)}{\partial I_2(t)} = -2c_4 (I_2^0(t) - \bar{I}_2) = -\dot{\varphi}_2(t) \quad (9)$$

$$\frac{\partial H(P_1^0(t), I_1^0(t), P_2^0(t), I_2^0(t), t)}{\partial P_1(t)} = -2c_1 (P_1^0(t) - \bar{P}_1) + \varphi_1(t) = 0 \quad (10)$$

$$\frac{\partial H(P_1^0(t), I_1^0(t), P_2^0(t), I_2^0(t), t)}{\partial P_2(t)} = -2c_3 (P_2^0(t) - \bar{P}_2) + \varphi_2(t) = 0 \quad (11)$$

From Equation (8)-(11), we obtain the following equations:

$$P_1(t) = \frac{1}{2c_1} \varphi_1(t) + \bar{P}_1 \quad (12)$$

$$P_2(t) = \frac{1}{2c_3} \varphi_2(t) + \bar{P}_2 \quad (13)$$

$$\varphi_1(t) = 2c_1(P_1(t) - \bar{P}_1) \quad (14)$$

$$\varphi_2(t) = 2c_3(P_2(t) - \bar{P}_2) \quad (15)$$

We can give the necessary and sufficient condition from Equation (12)-(15) as follows:

$$\begin{pmatrix} \dot{I}_1^0 \\ \dot{I}_2^0 \\ \dot{P}_1^0 \\ \dot{P}_2^0 \end{pmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{c_2}{c_1} & 0 & 0 & 0 \\ 0 & \frac{c_4}{c_3} & 0 & 0 \end{bmatrix} \begin{pmatrix} I_1^0 \\ I_2^0 \\ P_1^0 \\ P_2^0 \end{pmatrix} - \begin{pmatrix} q_1 \\ q_2 \\ \frac{1}{c_1} \bar{I}_1 \\ \frac{1}{c_3} \bar{I}_2 \end{pmatrix} \quad (16)$$

The boundary conditions are as follows:

$$\begin{pmatrix} I_1^0(0) \\ I_2^0(0) \end{pmatrix} = \begin{pmatrix} \bar{I}_1 \\ \bar{I}_2 \end{pmatrix} \quad (17)$$

$$\begin{pmatrix} P_1^0(0) \\ P_2^0(0) \end{pmatrix} = \begin{pmatrix} \bar{P}_1 \\ \bar{P}_2 \end{pmatrix} \quad (18)$$

Optimal solution can be formalized as follows:

$$\begin{aligned} f &= \int_0^T \{ (c_2 - 1) \bar{I}_1^2 t^2 + c_2 (\bar{P}_1 - q_1)^2 c_1 t^2 + (c_4 - 1) \bar{I}_2^2 t^2 + c_4 (\bar{P}_2 - q_2)^2 t^2 \} dt \\ &= \frac{1}{3} T^3 \{ (c_2 - 1) \bar{I}_1^2 + c_2 (\bar{P}_1 - q_1)^2 + (c_4 - 1) \bar{I}_2^2 + c_4 (\bar{P}_2 - q_2)^2 \} \end{aligned} \quad (19)$$

Let:

$$C_1 = \frac{1}{3} T^3 \{ (c_2 - 1) \bar{I}_1^2 + c_2 (\bar{P}_1 - q_1)^2 \} \quad (20)$$

$$C_2 = \frac{1}{3} T^3 \{ (c_4 - 1) \bar{I}_2^2 + c_4 (\bar{P}_2 - q_2)^2 \} \quad (21)$$

C_1 and C_2 are cost of manufacturing and cost of assembly process respectively.

From the Equation (19), in the assumption that there is a single CODP in the two-stage supply chain, the optimization model result is irrelevant to C_1 and C_3 , which shows production rate is the target production rate in the process of manufacturing and assembly when the total cost is minimum.

By solving Equation (3), we obtain $I_2(t)$ as follows:

$$I_2(t) = \int_{T_0}^T (\bar{P}_2 - q_2) dt = (\bar{P}_2 - q_2)(T - T_0) \quad (22)$$

By solving Equation (2), we obtain $I_1(t)$ as follows:

$$I_1(t) = \int_0^{T_0} (\bar{P}_1 - q_1) dt = (\bar{P}_1 - q_1)T_0 \quad (23)$$

When $\bar{I}_1 \neq 0$ and $\bar{I}_2 \neq 0$, optimal solution is as follows:

$$\begin{aligned} f &= \frac{1}{3}T^3 \{c_2(\bar{P}_1 - q_1)^2 + (c_2 - 1)I_1^2(t) + (c_4 - 1)I_2^2(t) + c_4(\bar{P}_2 - q_2)^2\} \\ &= \frac{1}{3}T^3 \{c_2(\bar{P}_1 - q_1)^2 + (c_2 - 1)(\bar{P}_1 - q_1)^2 T_0^2 + \\ &\quad (c_4 - 1)(\bar{P}_2 - q_2)^2 (T - T_0)^2 + c_4(\bar{P}_2 - q_2)^2\} \end{aligned} \quad (24)$$

Let:

$$\frac{\partial f}{\partial T_0} = 0$$

We can get location of CODP can be found as follows:

$$\begin{aligned} T_0^* &= \frac{(c_4 - 1)^2 (\bar{P}_2 - q_2)^2 T}{(c_2 - 1)(\bar{P}_1 - q_1)^2 + (c_4 - 1)^2 (\bar{P}_2 - q_2)^2}, \\ &\quad (c_2 \neq 1, c_4 \neq 1, \bar{P}_1 \neq q_1, \bar{P}_2 \neq q_2) \end{aligned} \quad (25)$$

Let:

$$\alpha = (c_2 - 1)(\bar{P}_1 - q_1)^2, \beta = (c_4 - 1)^2 (\bar{P}_2 - q_2)^2$$

Equation (25) can be expressed as follows:

$$T_0^* = \frac{\beta}{\alpha + \beta} T \quad (26)$$

From Equation (26), we can find under non-zero-inventory policy, the location of CODP in the product life cycle is proportional relevant to ratio of inventory cost of customization stage and total cost of supply chain.

3. Results and Analysis

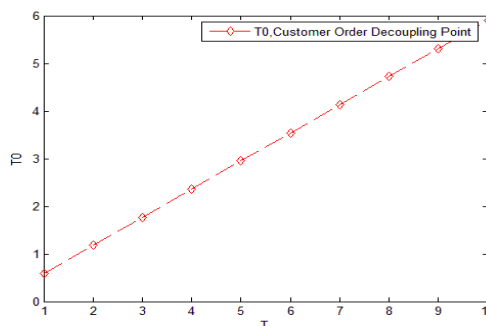


Figure 3. Relationship between PLC (product life cycle) and CODP

We assume that target production rate and demand rate are constants based on non-zero-inventory policy, and the data of parameters are $c_2=0.5$, $\bar{P}_1=0.5$, $c_4=0.4$, $q_2=0.6$. Results of model is shown in Figure 3.

From the curve of simulation results in the Figure 3, we can find the CODP of the two-stage supply chain under total optimal cost is located in the downstream position of the product life cycle, which satisfying the linear relation of Equation (26). It implies the CODP should be close to the terminal customer in order to achieve total optimal cost of the supply chain under non-zero-inventory. In this production mode, the cost advantage of the stage of mass production (manufacturing) has been fully embodied in the supply chain, and at this time, the total cost of supply chain reach a minimum combined with proper inventory policy. In view of the high-tech perishable products supply chain, which is also in line with the laws of supply and demand.

For testing results of Figure 3., we use the fmincon function in Matlab7.0, the model is converted into a quadratic planning function, and get the following results. According to the Matlab7.0, the two results are very close.

Table 1. The Location of CODP

PLC	1	2	3	4	5	6	7	8	9
CODP	0.62	1.37	1.98	2.45	3.12	3.52	4.23	4.87	5.45

4. Conclusion

In this paper we propose CODP positioning model of the two-stage supply chain aimed at high-tech perishable products based on the classic HMMS-type model. Using dynamic cost analysis, we do numerical simulations under different inventory strategies and production conditions. The results show that the model can reflect the numerical relation between production, inventory and CODP in the product life cycle and the whole supply chain. Under the optimal dynamic cost condition, CODP is located in the downstream of the supply chain. Relatively speaking, location of the products life cycle is more in front, production and inventory level is more stable; on the contrary, they have larger fluctuation. But the overall cost is rising in the supply chain with time. It also shows that the mass customization production system requires more sensitive reflect to market demand and personalized customization. Manufacturing chain is longer; its sensitivity is lower and cost corresponding is higher. Assembly chain is longer, its sensitivity is higher and the cost corresponding is lower.

This model only analyses single product and one-way logistics supply chain of high-tech perishable product. Because high-tech perishable product characteristics vary greatly, the next research can be focused on different categories of analysis and considering the recovery of perishable products.

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