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Noise Reduction of Chaotic Signal Based on Empirical Mode Decomposition

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Abstract

In view of the nonlinear characteristics of chaotic signal, a threshold denoising method of noisy chaotic signal based on empirical mode decomposition (EMD) is presented. Firstly, noisy chaotic signal is decomposed into several intrinsic mode function (IMF) by empirical mode decomposition. Secondly, the intrinsic mode functions of high frequency are respectively processed using threshold method. Finally, we add these IMFs with IMFs of low frequency to achieve denoising signal. The noisy Lorenz chaotic signal is chosen to perform noise reduction. Taking the noisy Lorenz chaotic signal as an example, the proposed method is used. The simulation results show that this method is an efficient method.

***Keywords**: the noisy chaotic signal, noise reduction, empirical mode decomposition*

1. Introduction

In the chaotic signal processing, the presence of noise has a severe impact on the calculation of system invariant parameter. Reference [1] points out that when calculating chaotic characteristic parameters such as fractal dimension, Lyapunov exponent, and so on, even if 1% of the noise, we calculate the dimension meaningless. Thus, the noise reduction effect will directly affect the result of chaotic signal processing.

Chaotic signal has broadband spectrum and the autocorrelation function of fast decay. Therefore the traditional filtering method not only can not remove the noise, but also will make the original chaotic signal distortion [1, 2]. This increases the complexity of signal, so the filter will be meaningless. In order to effectively reduce the noise interference, people have proposed some noise reduction methods, such as shadow noise reduction method [3], using the maximum likelihood estimation method of noise reduction [4], noise reduction of ship signals based on the local projective algorithm [5], self-adaptive decomposition level de-noising method based on wavelet transform [6], a quantum-inspired noise reduction method [7], An intelligent noise reduction method [8], and so on.

In view of the nonlinear characteristics of chaotic signal, a threshold denoising method of noisy chaotic signal based on empirical mode decomposition is presented in this paper. Firstly, chaotic signal polluted by noise is decomposed into several intrinsic mode function (IMF) by empirical mode decomposition. Secondly, the intrinsic mode functions of high frequency are processed using threshold method, and we add these IMFs with IMFs of low frequency to achieve denoising signal. The noisy Lorenz chaotic signal is chosen to perform noise reduction. Simulation results show that the proposed method is efficient to denoising noises and a much more cleaned signal can be obtained from this method.

2. Empirical Mode Decomposition

EMD [9], originally put forward by Huang et al. in 1998, is a signal processing method based on local characteristics of data in the time domain. It decomposes a signal into components of different frequencies by looking for local extrema so that different layers of the signal can be processed, respectively. It can be used to analyze nonlinear and non-stationary data. Different from common time-frequency analysis methods such as the short-time Fourier

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transform, the wavelet transform (WT), and so on, EMD is a totally data-driven method, independent of any basis functions.

EMD [10, 11] could be implemented as follows.

(1) Identify all the local maxima in the signal $x(t)$, and then connect them by a cubic spline as the upper envelope $x_{max}(t)$. Repeat the procedure for the local minima to produce the lower envelope $x_{min}(t)$.

(2) The mean $m_1(t)$ is calculated by using the equation as:

$$m_1(t) = [x_{max}(t) + x_{min}(t)]/2 \quad (1)$$

Calculate the difference between the $x(t)$ and $m_1(t)$:

$$h_1(t) = x(t) - m_1(t), \quad (2)$$

$h_1(t)$ is treated as the data, then:

$$h_{11}(t) = h_1(t) - m_{11}(t). \quad (3)$$

Repeat the sifting process k times until $h_{1k}(t)$ becomes an IMF which satisfies the IMF stop criterion.

$$h_{1k}(t) = h_{1(k-1)}(t) - m_{1k}(t), \quad (4)$$

Define $c_1(t) = h_{1k}(t)$, where $c_1(t)$ is the first IMF component from the data. A stop criterion can be accomplished by limiting the size of the standard deviation, SD, computed from the two consecutive sifting results. Here a typical value of SD can be set between 0.2 and 0.3.

$$\sum_{t=0}^{T} \frac{[h_{1(k-1)}(t) - h_{1k}(t)]^2}{h_{1(k-1)}^2(t)} < SD, \quad (5)$$

(3) The residual component after the first sifting process is:

$$r_1(t) = x(t) - c_i(t), \quad (6)$$

$r_1(t)$ is considered as a new signal and subjected to the same sifting process as described above.

$$r_n(t) = x(t) - \sum_{i=1}^{n} c_i(t), \quad (7)$$

Where $r_n(t)$ is the residue of the signal $x(t)$ and $c_i(t)$ stands for the ith IMF. The whole sifting process can be stopped by any of the following criteria: either when the component $c_n(t)$ or the residue $r_n(t)$ becomes smaller than the predetermined value, or when the residue $r_n(t)$ becomes a monotonic function from which no more IMF can be extracted. $x(t)$ can be express as:

$$x(t) = \sum_{i=1}^{n} c_i(t) + r_n(t). \quad (8)$$

Each IMF has to meet the following two conditions:

(i) The difference between the number of extremes and the zero crossings must either be less or equal than one point.

(ii) For any point of the signal, the mean value between the upper envelope and the lower envelope is zero.

3. A Threshold Denoising Method of Noisy Chaotic Signal Based on EMD

In the frequency domain, reference [12] studies the relationship of the instantaneous frequency for each IMF by EMD: (i) The first IMF has the highest instantaneous frequency component. (ii) The instantaneous frequency of the i-th IMF is twice of the i+1-th IMF almost everywhere. Therefore, each IMF can be regarded as a bandpass filtering for the original signal by EMD. In view of the above, Reference [13] introduces empirical mode decomposition (EMD) to interferogram filtering. With the first two intrinsic mode functions (IMFs) removed, phase noise can be reduced to some extent, but the detailed information is liable to be lost.

Based on the above, a threshold denoising method based on EMD shown in Figure 1 is proposed. Firstly, noisy signal is decomposed into several intrinsic mode function (IMF) by empirical mode decomposition. Secondly, the intrinsic mode functions of high frequency are respectively processed using threshold method. Finally, we add these IMFs with IMFs of low frequency to achieve denoising signal.

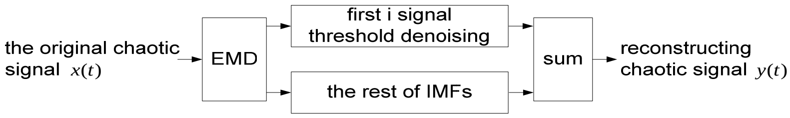


Figure 1. Diagram of Threshold Denoising Method with EMD

4. Example of Noise Reduction of Chaotic Signal

Lorenz model is as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a & a & 0 \\ c & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ -xz \\ xy \end{bmatrix}. \qquad (9)$$

Taking parameters a = 10, b = 3, c = 28, and using Runge-Kutta method to calculate the integral whose step is 0.01, and removing the unstable transition processes initially, we get x component. Using the x component signal whose data length is 1000 and adding Gaussian white noise, we get noisy Lorenz signal whose SNR is respectively -10dB and 0dB. The noisy Lorenz signal is shown in Figure 2 and Figure 5. IMFs derived from the noisy Lorenz signal by the EMD method is shown in Fig. 3 and Fig. 6. It is obvious that the number of high frequency IMF is 3 from Figure 3 and Figure 6. The IMF1~IMF3 are respectively processed using threshold method. Then we add these IMFs with IMFs of low frequency to achieve denoising signal shown in Figure 4 and Figure 7. Comparing Figure 2 and Figure 4, Figure 5 and Figure 7, we find that the proposed method is an efficient method.

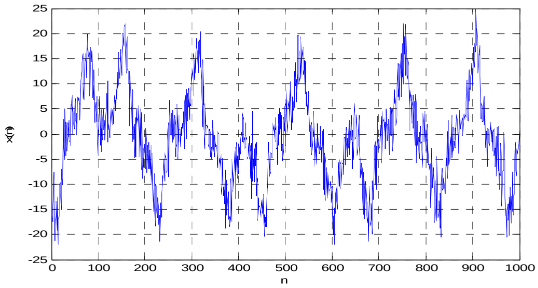
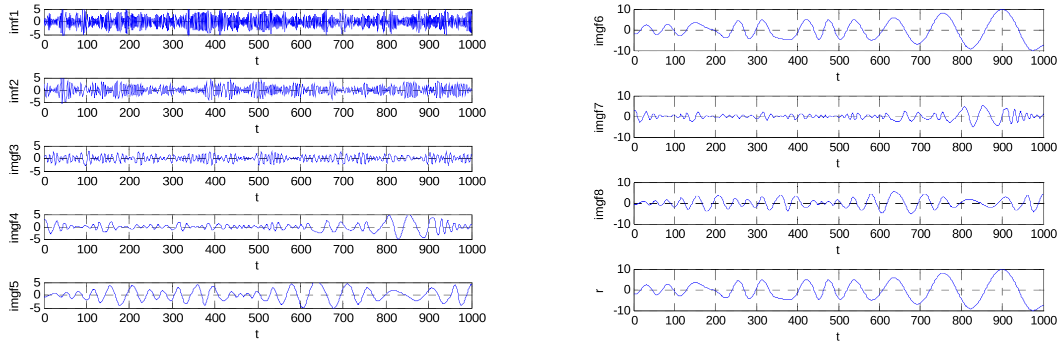


Figure 2. The Time-domain Waveform Before Noise Reduction when SNR is -10dB



(a) IMF1~ IMF5 (b) IMF6~ IMF8 and residue component r

Figure 3. IMFs Derived from the Noisy Lorenz Signal by the EMD Method when SNR is -10dB

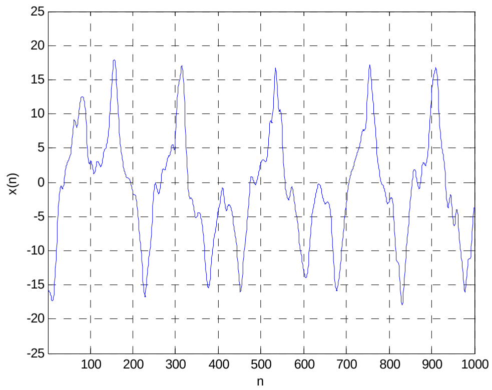


Figure 4. The Time-domain Waveform After Noise Reduction when SNR is -10dB

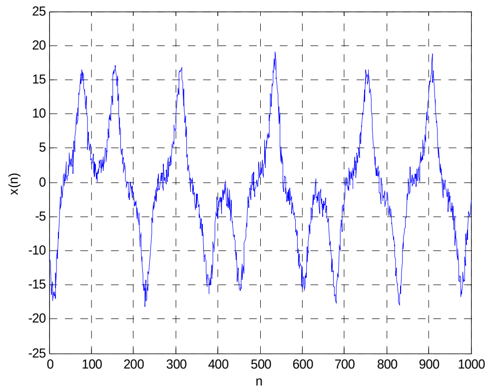
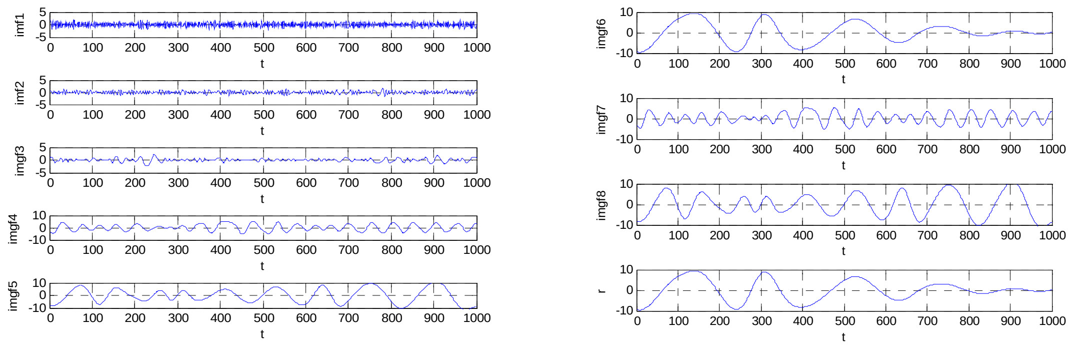


Figure 5. The Time-domain Waveform Before Noise Reduction when SNR is 0dB



(a) IMF1~ IMF5 (b) IMF6~ IMF8 and residue component r

Figure 6. IMFs Derived from the Noisy Lorenz Signal by the EMD Method when SNR is 0dB

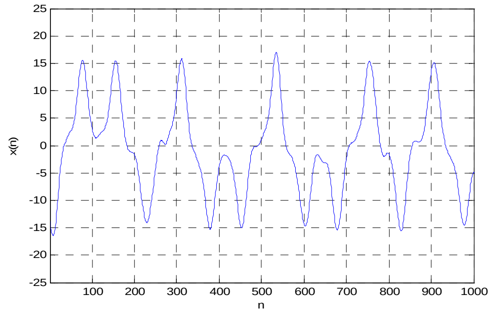


Figure 7. The Time-domain Waveform After Noise Reduction when SNR is 0dB

5. Quantitative Analysis of the Result

In order to quantitatively analyze the effect of noise reduction by using threshold denoising method based on EMD, we describe the effect of noise reduction by using the amount of SNR improvement before and after noise reduction which is proposed in reference [14]. The traditional definition of SNR is as follows:

$$\mathrm{SNR} = 20\log_{10}\frac{\sigma_s}{\sigma_n}, \quad (10)$$

Where σ_s and σ_n respectively represent the standard deviation of the signal and noise.

The amount of SNR improvement between the original time series x_1, x_2, …, x_n and the time series after the noise reduction y_1, y_2, …, y_n is defined as:

$$\Delta\mathrm{SNR} = 20\log_{10}\frac{\sigma_y}{\sigma_{x-y}}, \quad (11)$$

Where σ_y and σ_{x-y} respectively represent the standard deviation of the time series y_1, y_2, …, y_n after the noise reduction and the noise sequence x_1-y_1, x_2-y_2, …, x_n-y_n. The variance of the time series after the noise reduction and the variance of the noise are shown in Table 1. It can be seen from Table 1 that noise reduction effect for noisy chaotic signal by using empirical mode decomposition noise is obvious.

Table 1. The Variance of the Time Series After the Noise Reduction and the Variance of the Noise

SNR	σ_y	σ_{x-y}	ΔSNR
-10dB	68.9734	8.9102	17.8 dB
0dB	69.7438	1.2778	34.7 dB

6. Conclusion

Noise reduction of noisy chaotic signal is chaotic signal processing foundation. A threshold denoising method based on EMD is proposed. The noisy Lorenz chaotic signal is chosen to perform noise reduction. We obtained satisfactory results by calculating the amount of SNR improvement before and after noise reduction. Simulation results show that the proposed method is efficient to denoising noises and a much more cleaned signal can be obtained from this method.

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