An Improved Reconstruction Algorithm Based on Compressed Sensing for Power Quality Analysis in Wireless Sensor Networks of Smart Grid

Yi Zhong^{*1}, Jiahou Huang²

School of Information Engineering, Wuhan University of Technology, Wuhan, Hubei, P. R. China *Corresponding author, e-mail: zhongyi@whut.edu.cn¹, hjh1990114@whut.edu.cn²

Abstract

In recent years, the growing power quality problems in smart grid cause widespread concern at home and abroad. Because the traditional power quality algorithms which are based on Nyquist sampling theory have the drawbacks of complicated, heavy computations and poor real-time performance when sampling and analyzing continuous massive signals in smart grid. This paper discussed an improved reconstruction algorithm based on compressed sensing due to the sparsity of power quality signals in frequency domain for power quality analysis. By using the ZigBee wireless gateway for wireless sensor networks and energy metering chip, we develop a single meter node to do relative experiments. In the condition of the real test-bed and several compared experiments, power quality information in the highly compression ratio has good performance according to CSR (Compression Sampling Ratio), SNR (Signal to Noise Ratio), MSE (Mean Squared Error) and ERP (Energy Recovery Percentage), and will be widely used in power quality analysis.

Keywords: compressed sensing (CS), power quality (PQ), wireless sensor networks

Copyright © 2014 Institute of Advanced Engineering and Science. All rights reserved.

1. Introduction

In recent years, the growing power quality problems in single phase power grid cause widespread concern at home and abroad. The power quality problems mainly lie in several aspects: Power load in single-phase power grid is becoming more and more complicated and diversified. Modern electrical equipments, which are adopted for the sake of improving production efficiency, saving energy and decreasing pollution of the environment, are becoming the main resource of power quality problems. The single-phase power load which has the feature of nonlinear, rich harmonic, impactive and unbalanced will influence power grid and causes new problems of power quality. Power customers have increasingly demand of reliable power supply. Most precision electronics equipments and power electronics equipments which are controlled by computers and microprocessors are sensitive to the quality of power supply. Electronics equipments are more sensitive to the influence of power system than electromechanical equipments and demand for high requirements for power quality. Once the power grid appears problems, harms rang from economic losses to endangered power grid, equipments and personal safety, even for the community unstable which may affect social stability. Next-generation smart grid is composed of a great many discrete power generating, transmitting and distributing equipments. Many problems such as voltage bias fluctuation of regional power grids, harmonic pollution and increasing of reactive power factor are caused by the parallel operation of more and more small power generating equipments such as solar generators, wind turbines and thermal power generators. In order to ensure the safe and economic operation of smart grid and keep the stability and self-healing of power quality in microgrid, the research of power harmonic suppression and reactive compensation is becoming more and more urgent.

2. Related Works

2.1. Introduction of Compressed Sensing

In 2006, David L. Donoho et al proposed CS (Compressed Sensing) theory [8, 9], that

sparse signal with suitable reconstruction algorithm can be recovered from a very small set of measurements that far fewer than conventional measurement limited by Nyquist theorem. According to CS, it can sample and compress PQ information synchronously without any prior knowledge. Generally, CS theory basically consists of three steps: finding the sparsest decomposition of a signal, designing applicable compression representing matrix, which well approximates the original signal x in least coefficient, designing corresponding reconstruction algorithm, which reconstructs original signal length in N from observed M coefficients.

According to the theory, if original signal is sparse or in transform domain the original signal is sparse, using appropriate optimization algorithms can reconstruct original signals through a few number of sampled signals and the number of sampled signals used in reconstruction can be far below the number of sampled signals in the algorithms based on Nyquist theorem. CS theory is not the overall denial of Nyquist theorem, but using the sparsity of signals to reconstruct original signals through fewer sampled signals than the algorithms based on Nyquist standards.

CS (Compressed Sensing) theory take advantages of the sparsity of signals and use suitable reconstruction algorithms to reconstruct original signals through a very small set of observed values that far fewer than the signals limited by Nyquist theorem. Compared with the previous algorithms based on Nyquist sampling theorem, CS theory has the following advantages:

(1) CS theory's sampling speed is far lower than Nyquist's. And CS theory does global observations rather than local sampling; what is more, each observed value contains part of effective information of the signal. In the meanwhile, CS theory uses different observe algorithms every time to ensure the observed values has fewer information redundancy. Compared with the transform coding in sparse basis, the coefficients' location is no longer so important.

(2) In the aspect of decoder, the decoder has high robustness to the missing information for the reason that the importance of each projection coefficient is the same and the lose of several coefficients will has fewer influence to the reconstruction of original signals.

(3) Combining compression with sampling, algorithms based on CS theory use fewer memory spaces and computing resources than traditional sampling algorithms. Applying the saved resources in the later processing will reduce the cost of sampling and transmission.

(4) CS theory can relieve the computation burden of hardware and leave the computations to computers in later process and achieve the same reconstructive effects of original signal with the traditional algorithms by using the powerful parallel processing abilities of computers while keeping costs low.

The amounts of sampled signals are greatly reduced by the applications based on CS theory, solving the problems in signal processing, transmission and storage. And those applications develope rapidly in recent years:

In the aspect of sparse representation of signal, literature ten and eleven [10, 11] shows that the Fourier coefficients, wavelet coefficients of smooth signal, total variation norm of bounded variation functions, the Gabor coefficients of oscillator signal and Curvelet coefficients of image signal which has discontinuous edges have enough sparsity. However, how to find or construct orthogonal basis for a class of signals in order to get the best sparse representation of the signals is the problem needed to be studied further.

In the aspect of measurement matrix, literature twelve [12] points out that under the premise of RIP (Restricted Isometry Property, RIP) principle, we should reduce the dimensions of measurement matrix while ensure the loss information of original signal is minimal. Nowadays the measurement matrixes applied in CS theory are: Gaussian random matrix [10], binary random matrix (Bounerlli matrix), Fourier random matrix [11], Hadamard matrix etc.

In the aspect of signal recovery algorithms which means reconstruct original signal length in N from observed M coefficients, literature thirteen and fourteen [13, 14], point out that typical recovery algorithms are BP (Basis Pursuit, BP) algorithm, interior point algorithm, conjugate gradient projection algorithm and iterative threshold algorithm etc. Other reconstruction algorithms are OMP (orthogonal matching pursuit OMP) algorithm, TV reconstruction algorithm and other improved algorithms.

ROMP (Regularized Orthogonal Matching Pursuit, ROMP) algorithm is another marked improvement algorithm in traditional matching pursuit algorithm. ROMP algorithm is developed from traditional matching pursuit algorithm MP algorithm [16] and OMP algorithm [17]. ROMP

algorithm is on the basis of OMP algorithm and use regularization method to select elements which can select several eligible elements to support set and reducing the time of power signal reconstruction .What is more, the reducing of time is at the expense of reconstruction quality and should know the sparsity at first.

The process of ROMP algorithm is to use regularization method to process biggest k inner products of sensing matrix Φ and residual y and select one required element from the k inner products to reconstruct original signal.

ROMP algorithm combines the regularization method with OMP algorithm to achieve the goal of selecting more elements in one iteration. ROMP algorithm can classify elements fast and select more elements in one iteration that is the reason why ROMP can get faster reconstruction speed than OMP algorithm. However, ROMP algorithm also has its own drawbacks. This article proposed a new algorithm which is based on ROMP algorithm but can achieve better performance in power quality analysis.

2.2. Similarity and Threshold Regularized Orthogonal Matching Pursuit

Traditional ROMP (Regularized Orthogonal Matching Pursuit, ROMP) algorithm is based on OMP (Orthogonal Matching Pursuit, OMP) algorithm and uses regularization method to select element. ROMP algorithm provides a new self-adaptive algorithm to achieve the goal of getting faster speed of classifying elements and reducing the time of signal reconstruction.

Regularization method is a method which classifies elements according to the energy level of elements. Regularization method is described as followed: a set $A = \{x_i \mid i \in I_N\}$ $I_N = \{1, 2, ..., N\}$ is the index set of x_i . Classifying all the elements follow the rule:

$$|x_m| \le 2 |x_n|, m, n \in I_k \tag{1}$$

And classifies index set *I* into several subsets $I_k = \{1, 2, 3..., k\}$ and selects maximum energy subset I_0 in the last, that is, $\|X_{I_0}\| = \max\{\|X_{I_k}\|, k = 1, 2, ..., K\}$. Regularization method can classify elements fast and select more elements in one iteration which brings ROMP algorithm faster reconstruction speed.

However, ROMP algorithm has its own shortcomings. The algorithm is unreasonable that each time the algorithm can only select one group which has the maximum total energy to the candidate set and leaves other groups which have similar energy with the maximum total energy to the next iteration. ROMP algorithm brings a lot of redundant computation which is a waste of resources and demands for higher performance of equipments which leads to more equipment costs. In view of the whole iterative process, the tasks which can be done in one iterative are divided into several iterations, wasting the time and resources and decreasing the efficiency.

This paper points out that the maximum total energy group and other groups which are similar to maximum total energy group and have similar energy with the maximum total energy group should be put into the candidate set in the same iteration.

This paper proposed STROMP (Similarity and Threshold Regularized Orthogonal Matching Pursuit) algorithm which based on ROMP (Regularized Orthogonal Matching Pursuit) algorithm. STROMP algorithm changes the rules of selecting elements.

The threshold parameter is a, energy is E, energy correlation is M and average energy correlation is S:

$$E = \frac{1}{N} \sqrt{\sum_{i=1}^{N} x_i^2}$$
⁽²⁾

Where x_i is the member of set $A = \{x_i \mid i \in 1, 2, 3..., N\}$, N is the total number of set A.

$$M_{i} = \langle E_{1}, E_{i} \rangle (i = 2, 3...P)$$
(3)

$$S = \frac{1}{p-1} \left(\sum_{j=2}^{p} \langle E_{1}, E_{j} \rangle \right)$$
(4)

Where E_i is the energy of different vector J_i .

Use regularization method to classify inner products of signal residual and sensing matrix groups $J_1, J_2, J_3, \dots, J_p$ by Calculate into several energy. the energy $E_1, E_2, \dots, E_p(E_1 > E_2 > \dots > E_p)$ of $J_1, J_2, J_3, \dots, J_p$ therefore all the groups which energy are above $a * E_1$ and the energy correlation $M_i(i = 2, 3, 4...p)$ is above the average energy correlation S are selected to the candidate set in the same iteration.



Figure 1. Flow Chart of Voltage/Current Signal Compressed Sensing

The steps of STROMP algorithm are as follows:

in one iteration, reducing the iteration times and avoiding unnecessary steps.

Inputs: Sensing signal y, sensing matrix Φ , sparsity k and threshold coefficient a In this way, the groups which would be selected in several iterations before are selected

ŀ)

5989

Initialization: Residual r0 = y, support set $F_0 = \Phi$, iterator t = 1.

Step 1:

Use regularization method to select k largest elements of inner product of residual and sensing matrix by absolute value. Mark the group $J_1, J_2, J_3..., J_p$ from largest to smallest by energy and the energy of $J_1, J_2, J_3..., J_p$ is:

$$E_1, E_2, \dots E_p(E_1 > E_2 > \dots > E_p)$$
.

Step 2:

Use *S* and $a^* E_1$ to classify the group $J_1, J_2, J_3, ..., J_p$ which means that the groups which energy are above $a^* E_1$ and the energy correlation *M* between the group and E_1 is above average energy correlation *S* are selected into the candidate set *J*.

Step 3:

Calculate the reconstruction signals by the least square method: $X_t = \operatorname{argmin}_x \left\| y - \Phi_{F_t} x \right\|_2$, and updating residual.

 $r_t = y - \Phi_{F_t} x_t \, .$

Step 4:

if n < 2k, then update the number of cycles t = t+1 and go to step 1 or exit the loop. Output: $x = \Phi_F^+ y$;

The range of the threshold coefficient $a \in (0,1]$, in the research stage of this paper for power quality, the reconstruction effect will be best when the threshold value a is 0.6.

According to the STROMP (Similarity and Threshold Regularized Orthogonal Matching Pursuit) algorithm, the flow of the reconstruction algorithm is shown in Figure 1:

3. Experiment and Performance Analysis 3.1. Research Criteria and Platform Design

There is no unified standard in power quality generally and IEC definition for power quality is that power quality is the physical characteristics of power supply device's not disturbing and interrupting user's using electricity under normal working condition. Measuring the voltage current and power in single phase power grid in real time is help to study and analyze the characteristics of power quality.

In stable condition of linear load, voltage and current signals are both sine waveforms in 50Hz theoretically. But in unstable condition of nonlinear load, they are affected distorted by some inductances, capacitances, or other nonlinear factors. PQ Harmonics have N*50Hz frequency affected signals [15]. Voltage and current signals mainly consist of periodic or quasiperiodic signals in practical condition, and it exists a lot of information redundancy in periods or between periods.

We will introduce several performance indexes: CSR (Compression Sampling Ratio), SNR (Signal to Noise Ratio), MSE (Mean Squared Error), and ERP (Energy Recovery Percentage) to objectively appraise the reconstructed results of PQ signals.

$$CSR = \frac{N_c}{N} \times 100\%$$
(5)
$$SNR = 10 \lg \left[\frac{\sum_{i=1}^{N} |f(i)|^2}{\sum_{i=1}^{N} |f(i) - \hat{f}(i)|^2} \right]$$
(6)

An Improved Reconstruction Algorithm Based on Compressed Sensing for Power... (Yi Zhong)

$$MSE = \frac{\sqrt{\sum_{i=1}^{N} \left[f(i) - \hat{f}(i) \right]^2}}{\sqrt{\sum_{i=1}^{N} \left[f(i) \right]^2}} \times 100\%$$
(7)

$$ESP = \frac{\sum_{i=1}^{N} \left[\hat{f}(i) \right]^{2}}{\sum_{i=1}^{N} \left[f(i) \right]^{2}}$$
(8)

Where N is total sampling number of original signals, N_c is the reserved sample number of signals after sparse sampling, and $\hat{f}(i)$ is the reconstructed signal.

Based on theoretical research purposes mentioned above, we establish a set of smart meter test-bed with CS measurement. In Figure 2, this test-bed platform consists of three wireless nodes as smart meters, ZigBee wireless gateway, Ethernet router and terminal data server PC. For a single meter node, energy metering chip ADE7878 measures all information of single-phase load with internal hardware signal circuit and obtains voltage/current value, active power, reactive power, apparent power and etc. It transmits sampling data to STM32 system with SPI interface. STM32 system realizes PQ data storage with multi-tasking operating system (uC-OS). In ZigBee wireless network, the measuring node transmits all of PQ information to PC server. And we can configure node's charging settings online with infrared remote controller. Its LCD display shows PQ parameters real-time dynamically.



Figure 2. Smart Meter Test-bed with CS Measurement

In order to compare the performances of MP algorithm, OMP algorithm, ROMP algorithm and STROMP algorithm, we do a lot of contrast experiments which is show form Figure 3 to Figure 14 and the performance indexes is show in Table 1.

Table 1. Statistical Recovery Parameters of Compressing Voltage/Current Data from Different Reconstruction Algorithm

SNR(dB)	MSE (%)	ERP (%)
27.8448	4.0528	97.0766
21.2720	8.6377	99.0027
26.8008	4.5704	97.3575
11.6741	26.0794	98,2753
	SNR(dB) 27.8448 21.2720 26.8008 11.6741	SNR(dB) MSE (%) 27.8448 4.0528 21.2720 8.6377 26.8008 4.5704 11.6741 26.0794

ISSN: 2302-4046

OMP algorithm	SNR(dB)	MSE (%)	ERP (%)
voltage of load 1	33.7290	2.0585	99.5871
current of load 1	21.7750	8.1518	99.2397
voltage of load 2	36.1172	1.5636	99.0415
current of load 2	19.4868	106087	99.6135
ROMP algorithm	SNR(dB)	MSE(%)	ERP(%)
voltage of load 1	37.2835	1.5902	99.9720
current of load 1	28.9687	3.5610	100.5346
voltage of load 2	35.9712	1.6672	99.8083
current of load 2	27.7210	4.1110	99.6759
STROMP algorithm	SNR(dB)	MSE(%)	ERP(%)
voltage of load 1	43.0996	0.6999	100.1646
current of load 1	38.3961	1.2028	100.2397
voltage of load 2	42.0181	0.7927	99.9847
current of load 2	37.2896	1.3662	99.9020







Figure 4. The FFT Comparison of Voltage Recovery Signal and Original Signal through Different Algorithms in Load 1: (a) is MP algorithm, (b) is OMP algorithm, (c) is ROMP algorithm and (d) is STPOMP algorithm



Figure 5. The Recover Error of Voltage Recovery Signal and Original Signal through Different Algorithms in Load 1: (a) is MP algorithm, (b) is OMP algorithm, (c) is ROMP algorithm and d is STPOMP algorithm

5993



Figure 6. Current Recovery Signal and Original Signal Comparison through Different Algorithms in Load 1: (a) is MP algorithm, (b) is OMP algorithm, (c) is ROMP algorithm and (d) is STPOMP algorithm



Figure 7. The FFT Comparison of Current Recovery Signal and Original Signal through Different Algorithms in Load 1: (a) is MP algorithm, (b) is OMP algorithm, (c) is ROMP algorithm and (d) is STPOMP algorithm.



Figure 8. The Recover Error of Current Recovery Signal and Original Signal through Different Algorithms in Load 1: (a) is MP algorithm, (b) is OMP algorithm, (c) is ROMP algorithm and (d) is STPOMP algorithm



Figure 9. Voltage Recovery Signal and Original Signal Comparison through Different Algorithms in Load 2: (a) is MP algorithm, (b) is OMP algorithm, (c) is ROMP algorithm and d is STPOMP algorithm



Figure 10. The FFT Comparison of Voltage Recovery Signal and Original Signal through Different Algorithms in Load 2: (a) is MP algorithm, (b) is OMP algorithm, (c) is ROMP algorithm and (d) is STPOMP algorithm



Figure 11. The Recover Error of Voltage Recovery Signal and Original Signal through Different Algorithms in Load 2: (a) is MP algorithm, (b) is OMP algorithm, (c) is ROMP algorithm and (d) is STPOMP algorithm



Figure 12. Current Recovery Signal and Original Signal Comparison through Different Algorithms in Load 2: (a) is MP algorithm, (b) is OMP algorithm, (c) is ROMP algorithm and (d) is STPOMP algorithm



Figure 13. The FFT Comparison of Current Recovery Signal and Original Signal through Different Algorithms in Load 2: (a) is MP algorithm, (b) is OMP algorithm, (c) is ROMP algorithm and (d) is STPOMP algorithm



Figure 14. The Recover Error of Current Recovery Signal and Original Signal through Different Algorithms in Load 2: (a) is MP algorithm, (b) is OMP algorithm, (c) is ROMP algorithm and d is STPOMP algorithm

3.2. Contrast Experiments of Different Algorithms

In different load conditions, original sampling data in single-phase gird is send to PC terminal, and is regarded as raw data for CS recovery analysis with MATLAB. To show recovery effects under various load conditions, chip ADE7878 in smart meter is adopted to fixed sampling rate at 4KHz with two different loads. Load 1:1K linear resistor; Load 2: 5W nonlinear switching power load.

According to the above experiments, we can see that no matter in which load, STROMP algorithm does much better than other classic matching pursuit algorithm. Contrast to the ROMP algorithm, STROMP algorithm does a great improvement in linear load, and has effect as great as ROMP algorithm in none linear load. Compare the original sampled signal with reconstruction signals, we can see that the reconstruction signal keep most of the harmonic components and each harmonic component is almost the same, which means that STROMP algorithm has great effect in the study and research of power quality.

4. Conclusion

This paper advances an improved reconstruction algorithm based on compressed sensing for power quality analysis in wireless sensor networks of smart grid. With the smart test-bed, real data is sparsely sampled and evaluated by STOMP algorithm. The sampling rate about voltage and current signals is up to 50% of the Nyquist's. The reconstruction SNR of linear load is more than 43dB, and that of nonlinear load is more than 42dB.

According to the experiments, STROMP algorithm reduces the data transmitted in the smart grid which can alleviate the burden of transmitting network and enhance the receive quality of signal. However, due to the different influences such as the limit of environment, the none linear load of single phase power grid of different customer and areas may increase the

difficulty of power quality measurements. In different environment we can get different performances.

5. Conflict of interest

The authors declared that they have no conflicts of interest to this work.

Acknowledgements

This work is supported by National Science Foundation of China under Grant No. 51175389, the National High Technology Research and Development Program (863 Program) of China under Grant No. 2012AA041203, and the Fundamental Research Funds for the Central Universities under Grant Nos. 2012-IV-090 and 2011-II-002.

References

- B Yunus, H Li. Analysis of power quality waveform for data transmission efficiency over IEC 61850 communication standard. First International Power and Energy Conference. 2006: 161-166.
- [2] Candes EJ, Tao T. Near optimal signal recovery from random projections: Universal encoding strategies. IEEE Trans. on Information Theory. 2006; 52(12): 5406-5425.
- [3] DONOHO DL. Compressed Sensing. IEEE Trans Information Theory. 2006; 52(4): 1289- 1306.
- [4] E Cand s, J Romberg, Terence Tao. Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information. *IEEE Trans. on Information Theory*. 2006; 52(2): 489-509.
- [5] E Cand s. The restricted isometry property and its implications for compressed sensing. Academic sciences. 2006; 346(I): 598-592.
- [6] EJ Candes. Compressive Sampling [A] Proceedings of the International Congress of Mathematicians. Madrid, Spain. 2006; 3: 1433- 14521.
- [7] Figue Iredo Mat, Nowakrd, Wrights JG. Radiant projection for sparse reconstruction: application to compressed sensing and other inverse problems. *IEEE J-STSP*. 2007; 1(4): 586-598.
- [8] H Xue, R Yang. Optimal Interpolating Windowed Discrete Fourier Transform Algorithms for Harmonic Analysis in Power Systems. IEEE Proceedings: Generation, Transmission and Distribution. 2003; 150: 583-587.
- [9] JA Tropp, AC Gilbert. Signal recovery from random measurements via orthogonal matching pursuit. *IEEE Transactions on Information Theory*. 2007; 53(12): 4655~4666.
- [10] J Barros, R1 Diego. Analysis of harmonics in power systems using the wavelet-packet transforms. *IEEE Transactions on Instrumentation and Measurement*. 2008; 57(1); 63-69.
- [11] NE Huang, Z Shen, SR Long et al. *The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis.* Proceedings of the Royal Society of London. 1998.
- [12] KKC Yu, NR Watson, J Arrillaga. An adaptive Kalman filter for dynamic harmonic state estimation and harmonic injection tracking. *IEEE Transactions on Power Delivery*. 2005; 20(2): 1577-1584.
- [13] Michael L, et al. Sparse MRI: The application of compressed sensing for rapid MRimaging. Available:http://www.stanford.edu/~mlustig/SparseMRI.pdf. 2007
- [14] NE Huang, Z Shen, SR Long, et al. The empirical mode decomposition and the Hilbert spectrum for nonlinear and non- stationary time series analysis. Proceedings of the Royal Society of London. 1998.
- [15] RA DeVore, VN Temlyakov. Some remarks on greedy algorithms. Advances in Computational Mathematics. 1996: 173~187.
- [16] SG Mallat, Z Zhang. Matching pursuits with time-frequency dictionaries. IEEE Transactions on Signal Processing. 1993; 41(12): 3397~341.
- [17] S Mishra, CN Bhende, BK Panigrahi. Detection and classification of power quality disturbances using s-transform and probabilistic neural network. *IEEE Transactions on Power Delivery*. 2008; 23(1): 280-287.
- [18] T Lobos, T Kozina, Z Leonowicz. High resolution spectrum estimation methods for signal analysis in power electronics and systems. Proceedings of the IEEE International Symposium on Circuits and Systems, Geneva. 2000; 2: 553-556.
- [19] Zhang L. Sparsity-based AOA Estimation for Emitter Localization. TELKOMNIKA Indonesian Journal of Electrical Engineering. 2012; 10(4): 769-774.
- [20] Xueming Z, Xiaobo Y, Wang D. Improved Compressed Sensing Matrixes for Insulator Leakage Current Data Compressing. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2014; 12(6).